JAMES COOK UNIVERSITY
AUSTRALIA

Maths Refresher

Working with Decimals

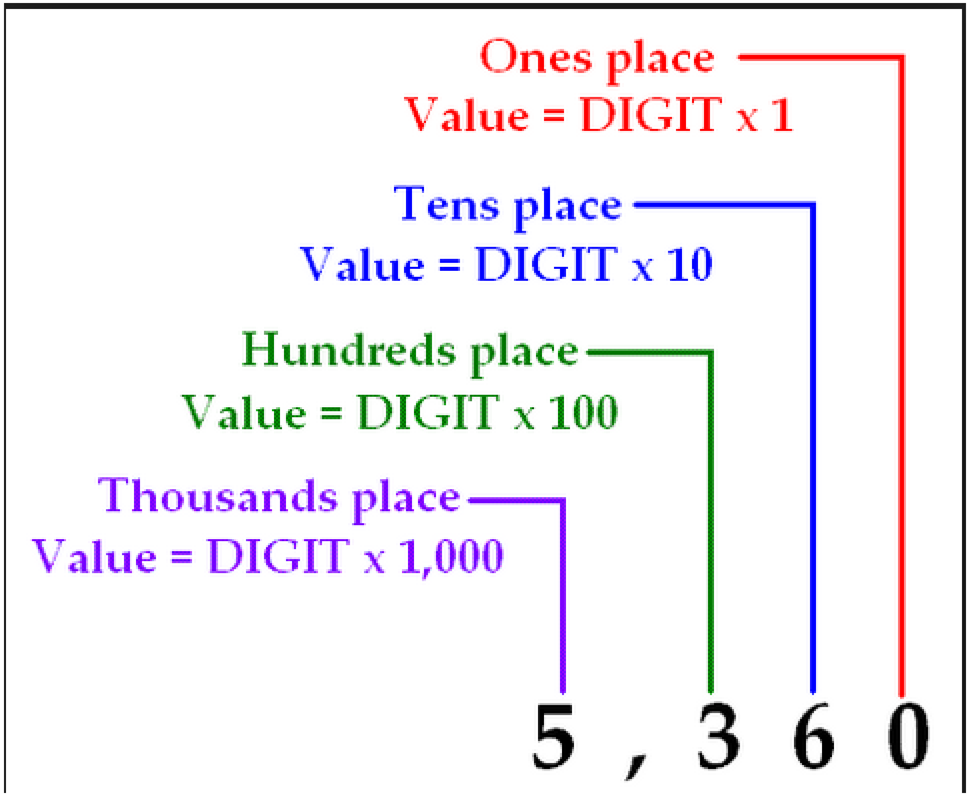
Working with Decimals

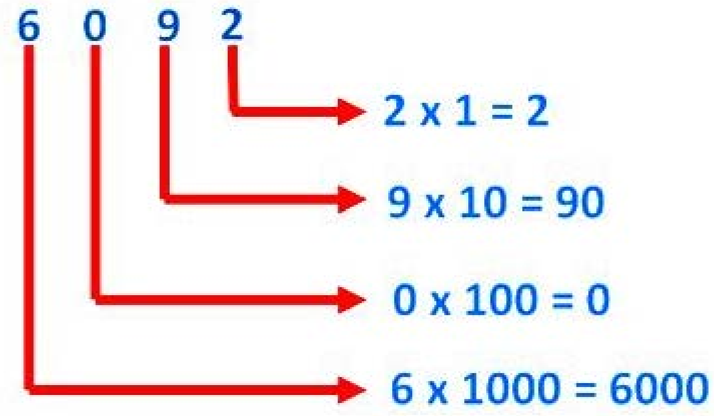
JAMES COOK UNIVERSITY
AUSTRALIA

Intent….

- The decimal separates whole numbers from parts of a whole.
- Each digit in a number has a ‘place value’
- The value depends on the position of the digit in that number
- Each position can be thought of as columns
- Each column is a power of ten.

Place value recap

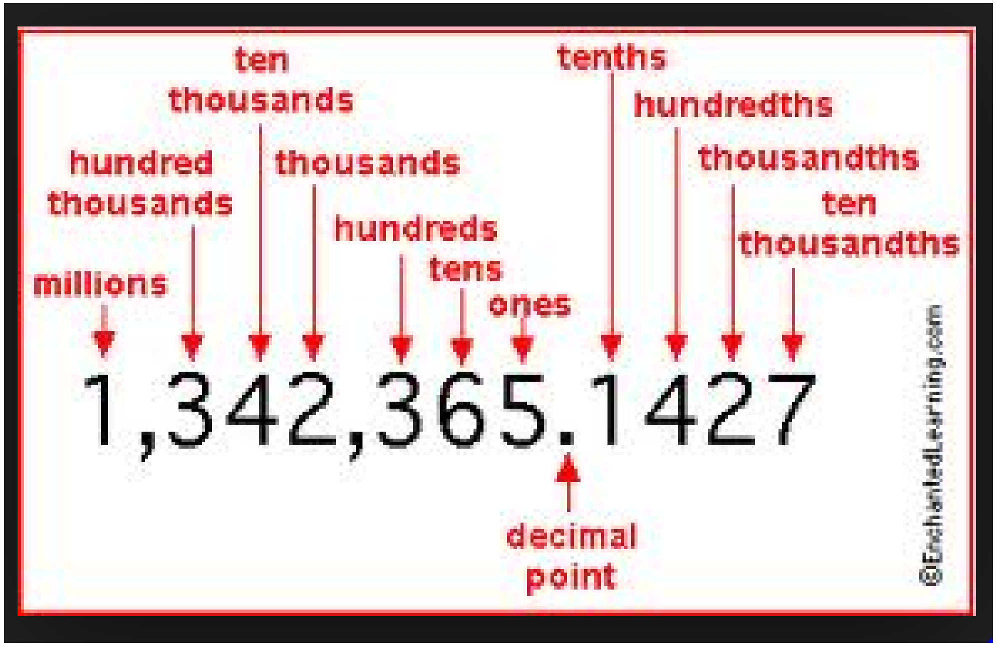




Working with Decimals

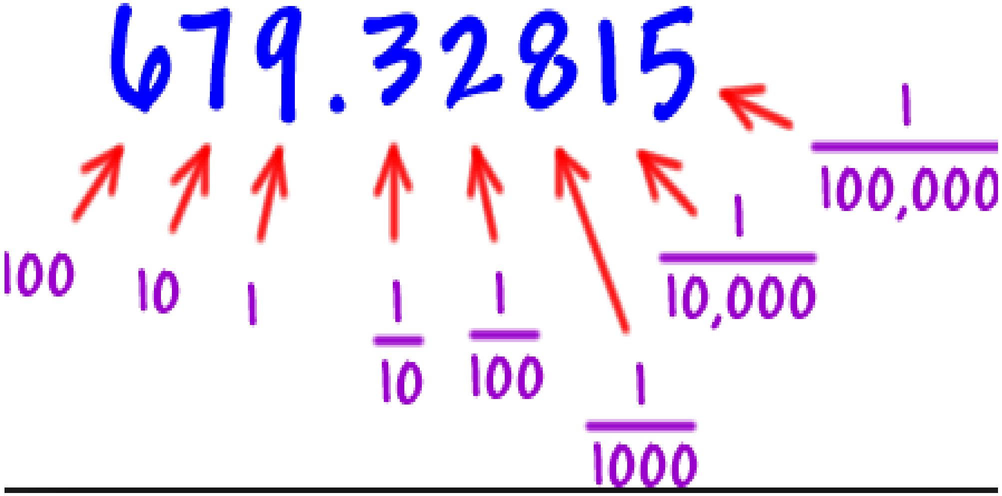
For example:

- The decimal indicates which digit is in the ‘ones’ place. Once this digit is known, we can determine the place value of all other digits in the number.



Working with Decimals

- A digit’s decimal place is its position to the right of the decimal.
- For example, in the numeral 679.32815, 3 is in the first decimal place, 2 is in the second and so on.



Powers of ten

Any number raised to the power of zero is one because when we divide numbers of the same base with a power, we subtract the power and get zero. We will explore this further in workshop five, when we investigate the ‘index laws

$10^4 = (10 \times 10 \times 10 \times 10)$

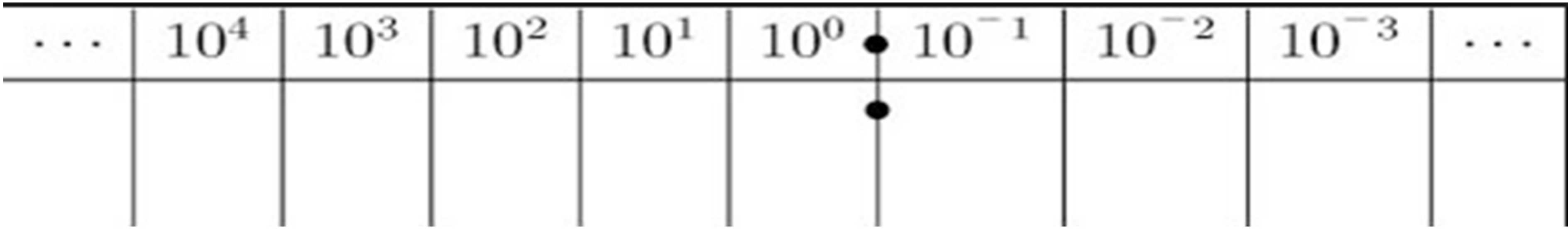
$10^3 = (10 \times 10 \times 10)$

$10^2 = (10 \times 10)$

$10^1 = (10)$

$10^0 = 1$

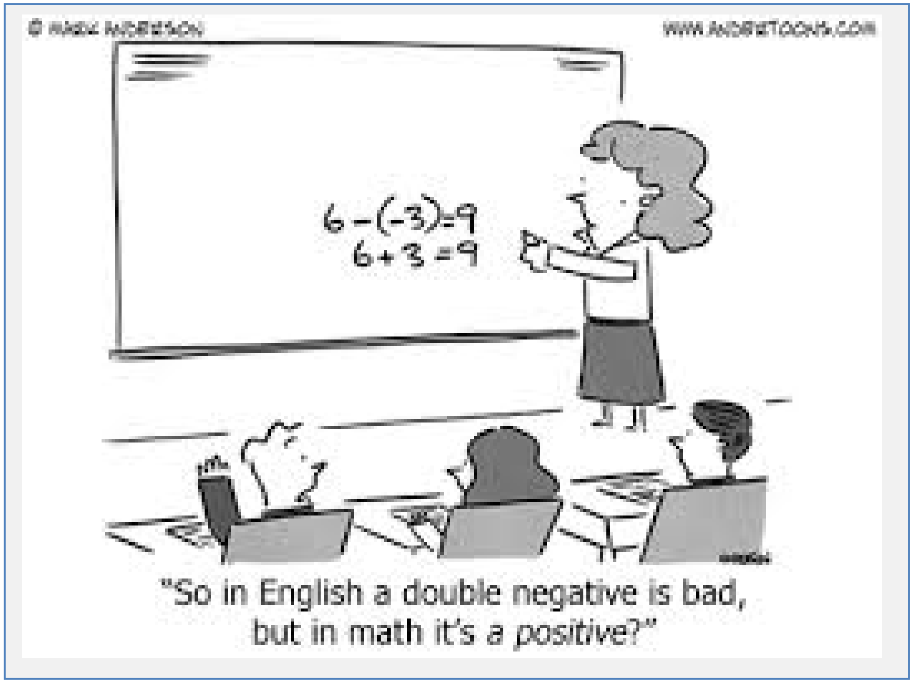
- $10^{-1} = (\frac{1}{10})$
- $10^{-2} = (\frac{1}{100})$
- $10^{-3} = (\frac{1}{1000})$



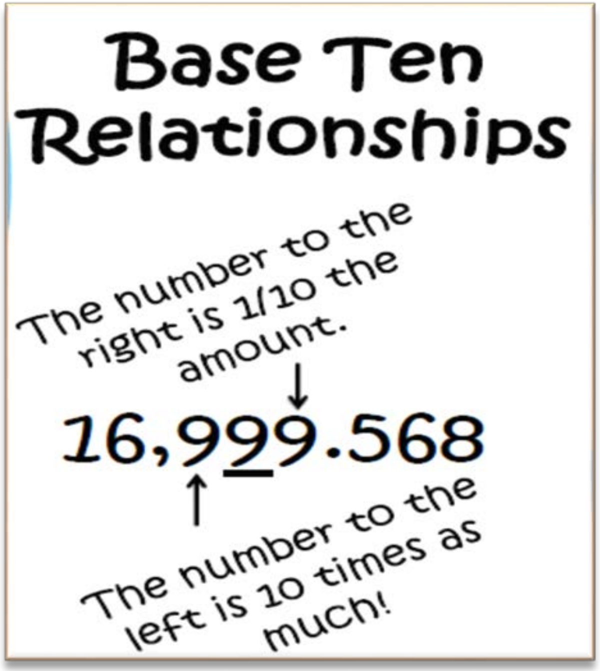
. . .	10^4	10^3	10^2	10^1	10^0 •	10^{-1}	10^{-2}	10^{-3}	. . .
					•				

Relationships

- Mathematics learning is easy when we see mathematical relationships



"So in English a double negative is bad, but in math it's a *positive*?"



https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/algebra-foundations-scientific-notation/v/scientific-notation

Decimal facts

- A recurring decimal is a decimal fraction where a digit repeats itself indefinitely
- For example, two thirds = 0.666666
- Because the number repeats itself from the tenths position a dot can be written above the 6 as such $0.\dot{6}$
- If the number was one sixth, 0.16666 we write $0.1\dot{6}$
- If the number contained a cluster of repeating digits, for example, five elevenths, 0.454545 we write $0.\dot{4}\dot{5}$
- A terminating decimal is a number terminates after a finite (not infinite) number of places, for example:
- $\frac{2}{5} = \frac{4}{10}$ or 0.4; and $\frac{3}{16}$= 0.1875 (it terminates after 5)

Working with Decimals

Rounding and significant figures

- When talking about a number such as 24.66666666
- It is complex to talk about it as rounding to the next hundredth, thousandth… too hard to say!!
- Instead we round to so many decimal places
- 25 when rounded to the next whole number
- 24.7when rounded to one decimal place
- 24.67 when rounded to two decimal places
- 24.667 when rounded to three decimal places

"Rounding decimals: to the nearest tenth"

https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/dec-rounding-estimation-pre-alg/v/rounding-decimals

Working with Decimals

- **Rounding and significant figures**
- We can work with a more complex idea such as rounding to so many ***significant figures (s.f)***
- If we talk about large amounts of money, for example, in a job we could have the potential to earn $17,632.31 or $17.672.36 hence the salary is approximately $17,600. The $32.31 and $72.36 are not really significant when talking about large amounts.
- However, for shoes the difference between $32 & $72 is significant.
- Thus, given the salaries above we can round to three significant figures: $17,600 and $17,700 respectively
- The term significant figures is abbreviated as s.f. they are non zero figures.
- 1694 can be rounded down to three s.f. so it will be 1690

Working with Decimals

Where do you see decimals in the real world?

- Money and measurement…..

The decimal is a separator

- Let’s think about 3.5; what does this number tell us?
- Think of this in terms of money $3.50 three whole units and fifty cents, or half of one dollar.
- In measurement, 3.5cm

Your turn…

- *Write 3.5cm in metres and in millimetres*

Working with Decimals

- On the number line mark 0.5
- Then mark 3.5
- Then convert 3.5 to metres and then millimetres

- 3.5cm is the same as 35mm or 0.035m

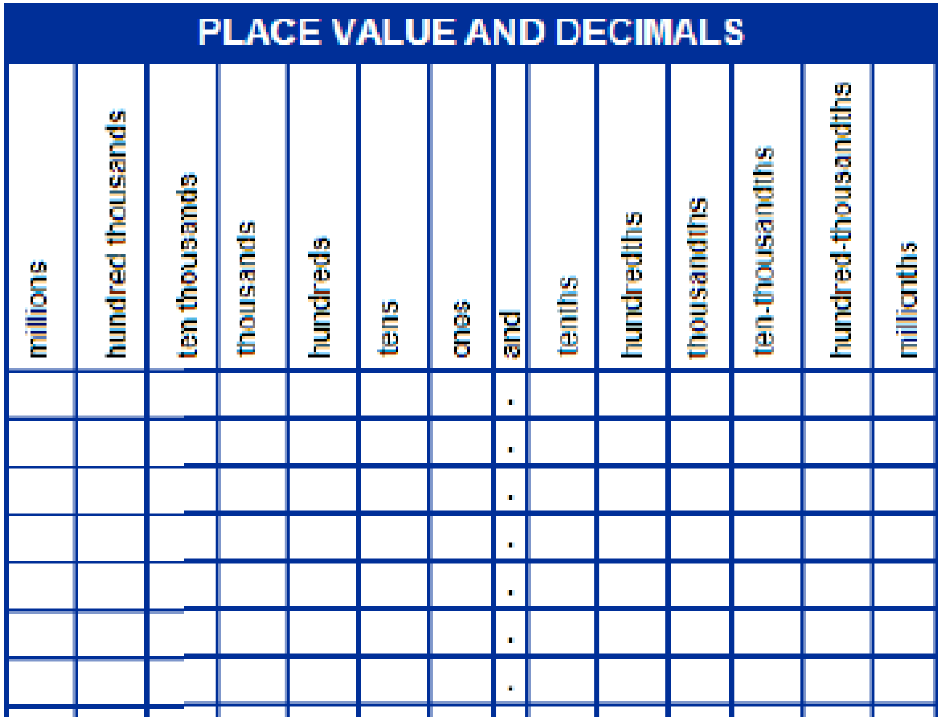
Working with

What happens when we multiply or divide by ten, or powers of ten?

- We understand patterns when multiplying by ten.
- However, often we say we add a zero. Are we correct?
- Think about 4.3 x 10, does it equal 4.30?
- Another misconception is that we move the decimal one place.
- Whereas, it is actually the digits that move.
- When we multiply by ten, all digits in the number become ten times larger and they move to the left.
- *What happens when we divide by 10?*

Your turn

- When we divide by ten, the digits in the number all move to the right….
- Practise:
- 865.32÷ 10=
- 865.32÷ 100 =
- 865.32× 10=
- 865.32× 100 =



PLACE VALUE AND DECIMALS

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	and	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
							.						
							.						
							.						
							.						
							.						
							.						
							.						

Working with Decimals

- The most common way we work with decimals in our daily lives is when we shop or work in retail.
- The key point when we work with addition or subtraction of numbers is to line up the decimal points.
- The zero will often be regarded as a place holder.
- For example, 65.32+74.634=

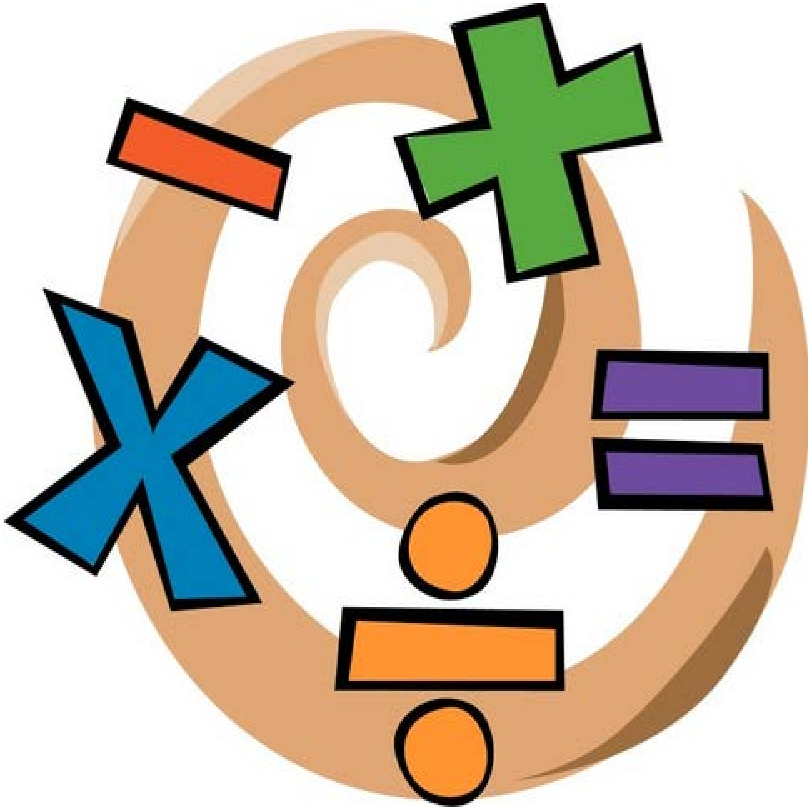
$$\begin{array}{rrl} & 65 & .320 \\ + & 74 & .634 \\ \hline & 139 & .954 \end{array}$$

Computation

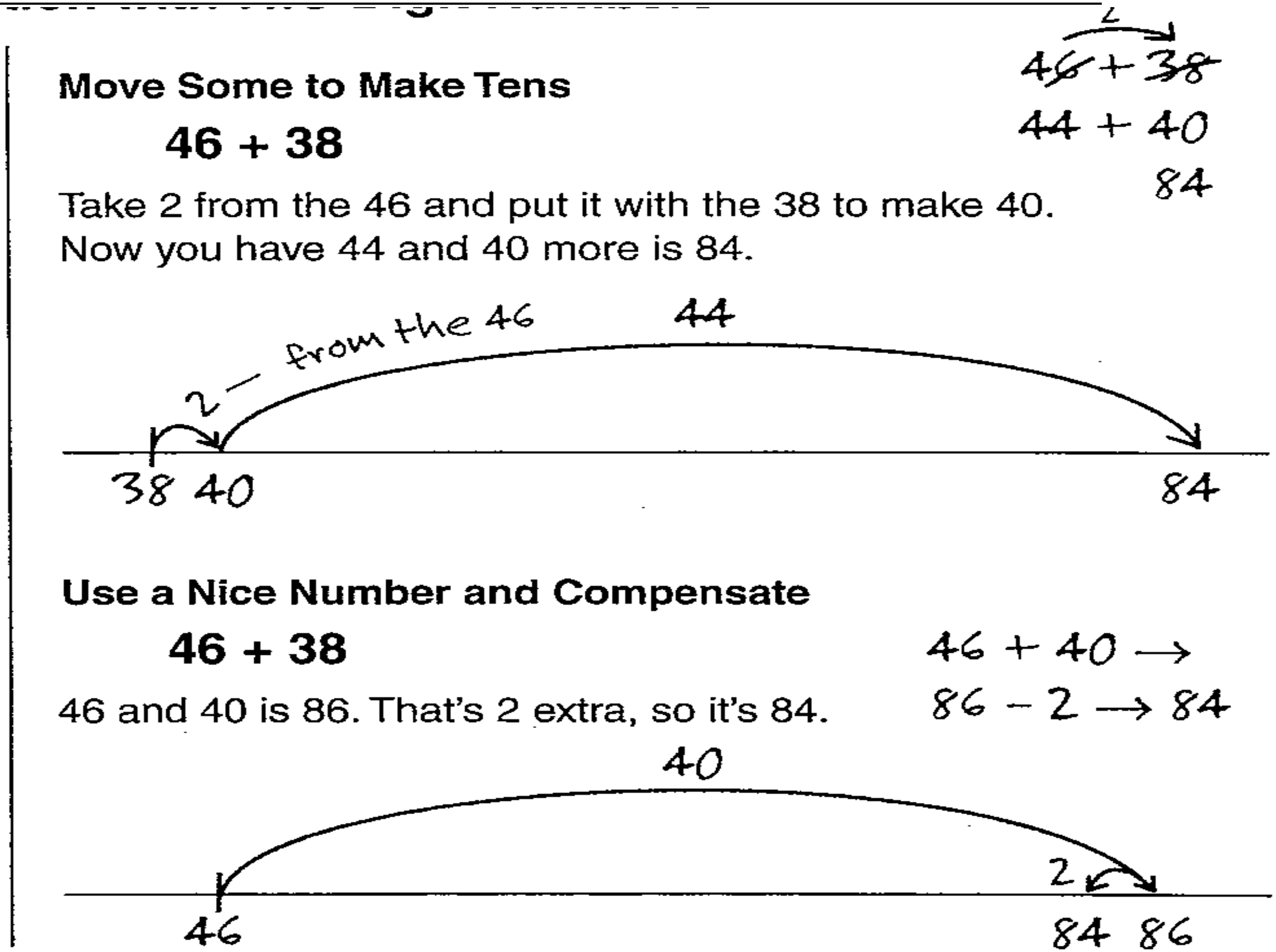
- We can now see that **decimals** are related to the **place value** concept
- In mathematics learning it is essential to develop deep understandings about the concept of place value.
- The following slides will be a revision on **computation** strategies and possibly provide you with new strategies to try.
- Often when working with larger numbers a process of **renaming** is required
- This renaming occurs when we trade, or **decompose** numbers…
- Graphics from: Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Sydney: Pearson Education.

Computation

- Following are some different strategies for you to explore in relation to place value, and the four arithmetic concepts



Compensate



Move Some to Make Tens

46 + 38

Take 2 from the 46 and put it with the 38 to make 40.
Now you have 44 and 40 more is 84.

$\overset{2}{\curvearrowright}$
$4\cancel{6} + 3\cancel{8}$
$44 + 40$
84

Use a Nice Number and Compensate

46 + 38

46 and 40 is 86. That's 2 extra, so it's 84.

$46 + 40 \rightarrow$
$86 - 2 \rightarrow 84$

Add to ten

Add Tens, Add Ones, Then Combine

46 + 38

40 and 30 is 70. 6 and 8 is 14.
70 and 14 is 84.

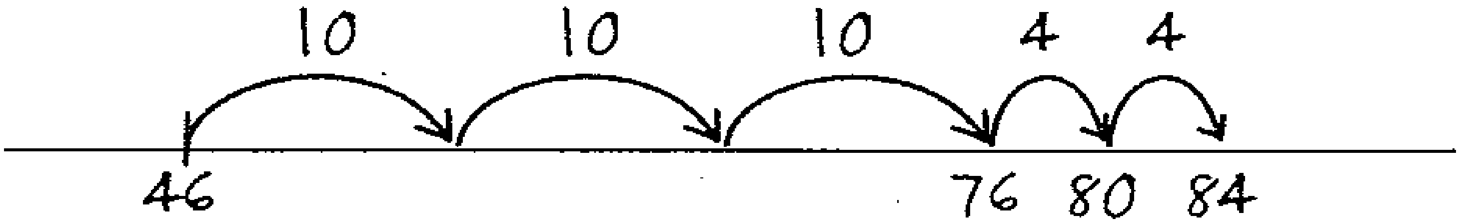
40 + 30 = 70
6 + 8 = 14
84

Add on Tens, Then Add Ones

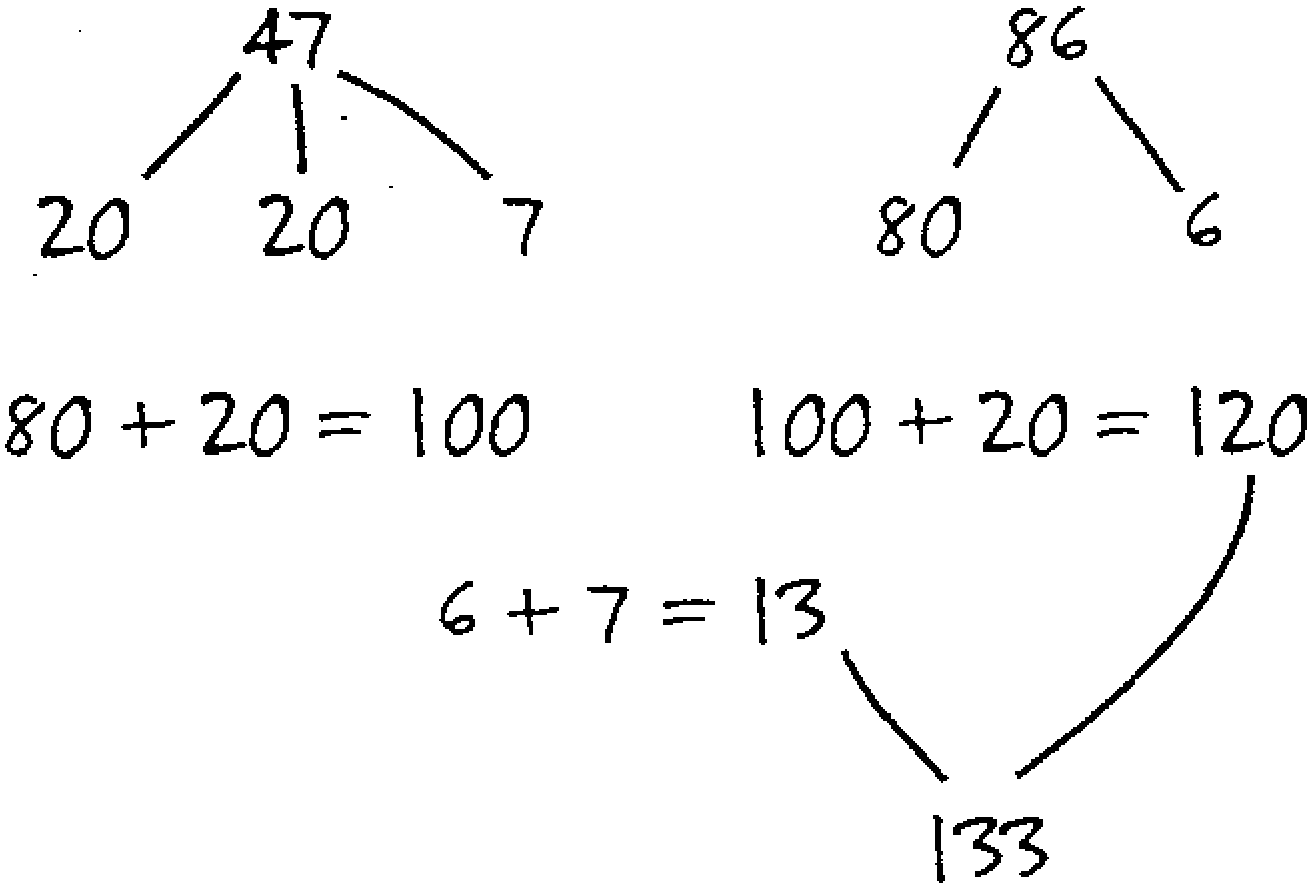
46 + 38

46 and 30 more is 76. Then I added on the other 8. 76 and 4 is 80 and 4 is 84.

46 + 30 →
76 + 8 → 80, 84



Example….. 47+86=133



80 + 20 = 100

100 + 20 = 120

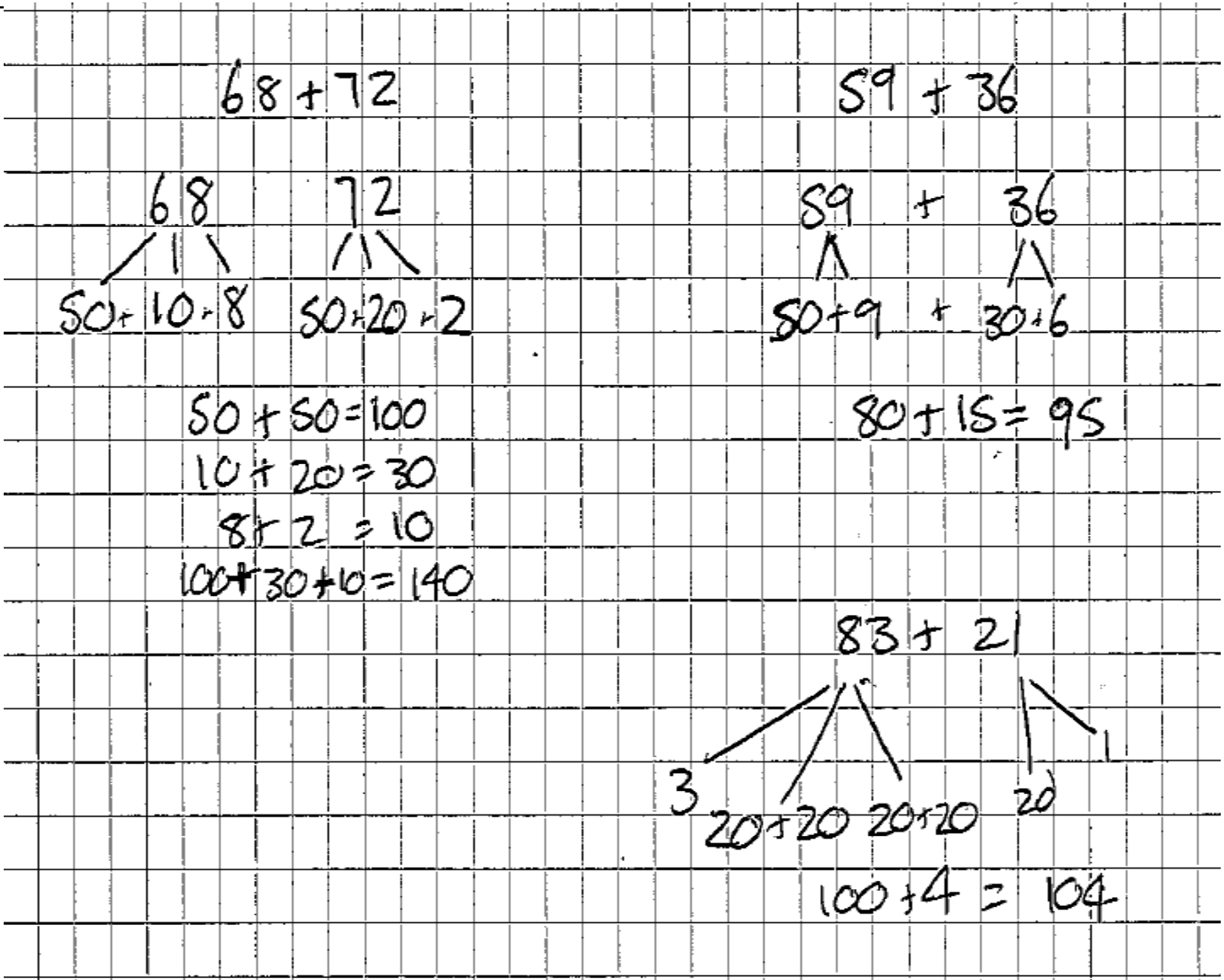
6 + 7 = 13

Your turn…

Your turn

- 68+72
- 59+36
- 83+21

Examples…



68+72

68 72

50+10+8 50+20+2

50+50=100

10+20=30

8+2 =10

100+30+10=140

59 + 36

59 + 36

50+9 + 30+6

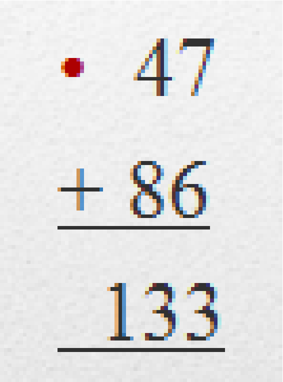
80+15= 95

83+ 21

3 20+20 20+20 20 1

100+4 = 104

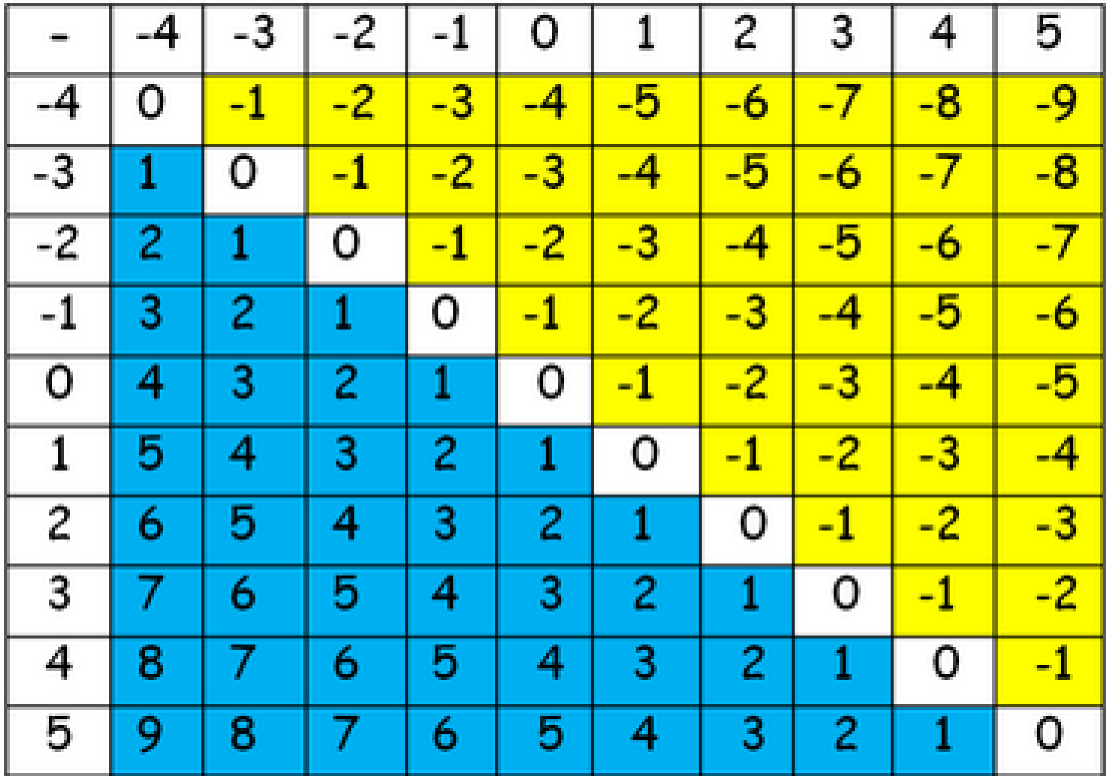
Example…. 47+86=133



• 47
+ 86
133

The first step is to add the ones and we get 13ones or 3 ones and 1 ten, so we add the 1ten to the tens then we add 5tens and 8 tens to get 13tens or 130+3

Subtraction



-	-4	-3	-2	-1	0	1	2	3	4	5
-4	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
-3	1	0	-1	-2	-3	-4	-5	-6	-7	-8
-2	2	1	0	-1	-2	-3	-4	-5	-6	-7
-1	3	2	1	0	-1	-2	-3	-4	-5	-6
0	4	3	2	1	0	-1	-2	-3	-4	-5
1	5	4	3	2	1	0	-1	-2	-3	-4
2	6	5	4	3	2	1	0	-1	-2	-3
3	7	6	5	4	3	2	1	0	-1	-2
4	8	7	6	5	4	3	2	1	0	-1
5	9	8	7	6	5	4	3	2	1	0

subtraction table of integers

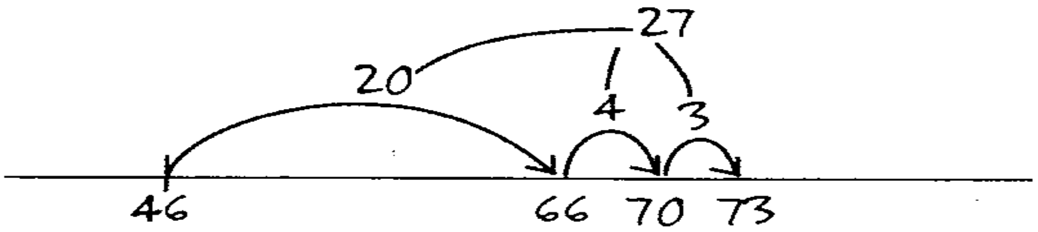
Subtraction

Add Tens to Get Close, Then Ones

73 – 46

46 and 20 is 66. (30 more is too much.) Then 4 more is 70 and 3 is 73. That's 20 and 7 or 27.

46 + 20 = 66
66 + 4 = 70
70 + 3 = 73
20 + 4 + 3 = 27

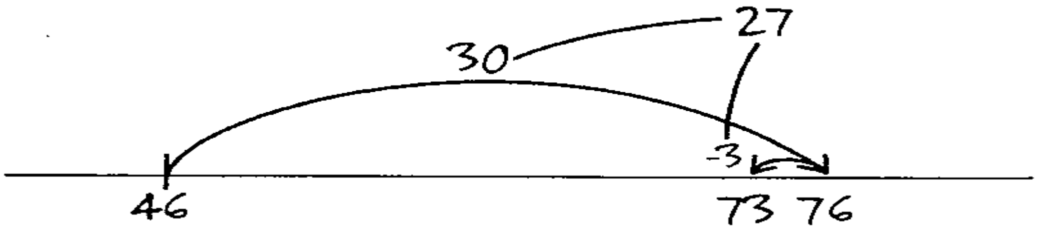


Add Tens to Overshoot, Then Come Back

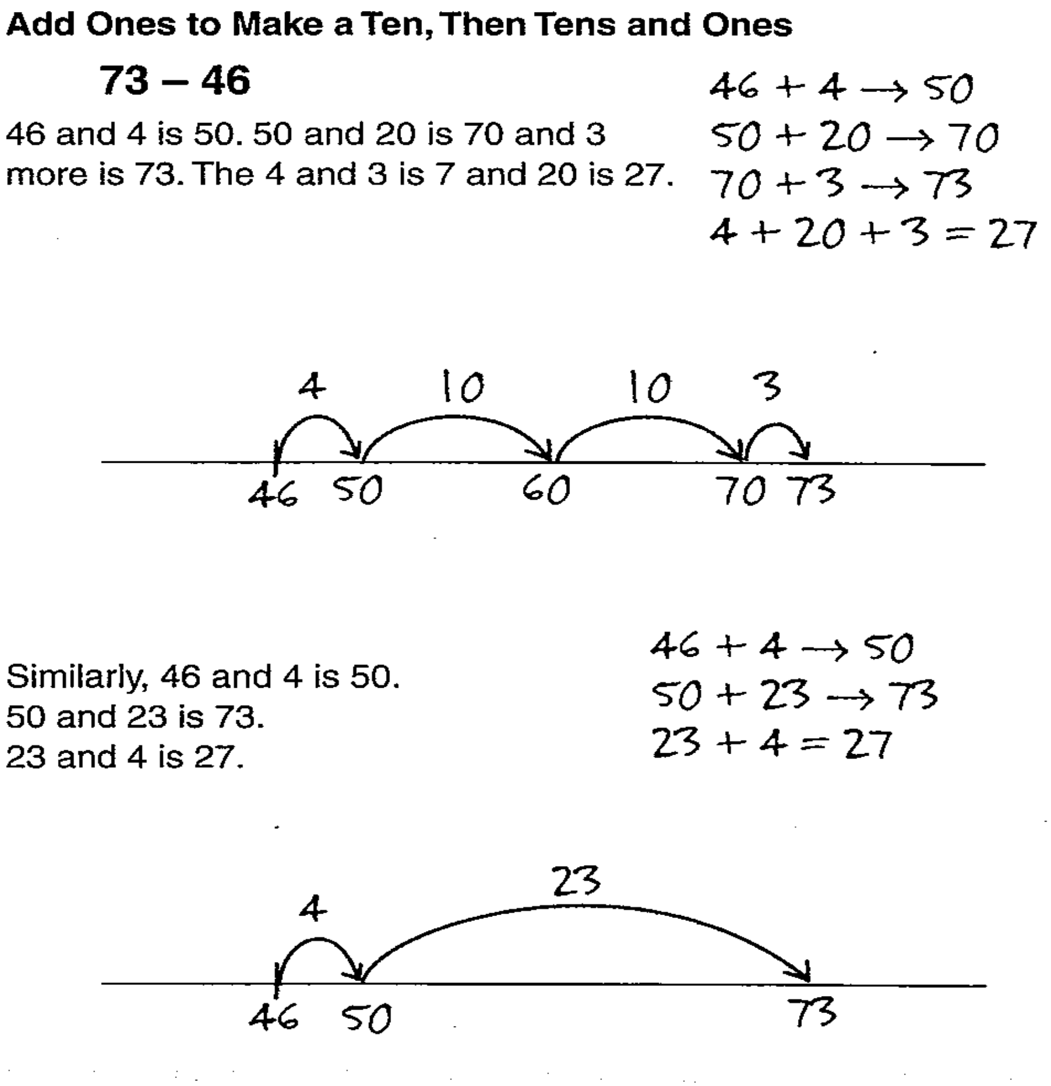
73 – 46

46 and 30 is 76. That's 3 too much, so it's 27.

$46 + 30 \rightarrow 76 - 3 \rightarrow 73$
$30 - 3 = 27$



Subtraction



Add Ones to Make a Ten, Then Tens and Ones

73 – 46

46 and 4 is 50. 50 and 20 is 70 and 3 more is 73. The 4 and 3 is 7 and 20 is 27.

$46 + 4 \rightarrow 50$

$50 + 20 \rightarrow 70$

$70 + 3 \rightarrow 73$

$4 + 20 + 3 = 27$

Similarly, 46 and 4 is 50.
50 and 23 is 73.
23 and 4 is 27.

$46 + 4 \rightarrow 50$

$50 + 23 \rightarrow 73$

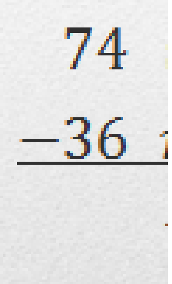
$23 + 4 = 27$

Subtraction …74-36=38

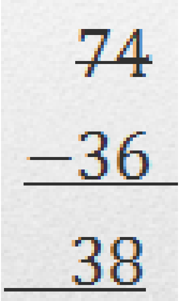
- $74 - 36 =$ we can read 74 as 7 *tens* and 4 *ones* or 6 *tens* and *14 ones.* This is called **decomposing** numbers.

6 14

74 so first we cannot take 6 from 4, so we decompose



74
−36



~~74~~
−36
38

—

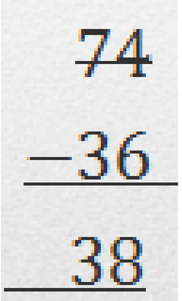
36 now we can subtract 6 *ones* from 14*ones*

then we take 3 *tens* from 6 *tens*

Subtraction ….. 74-36=38

- $74 - 36 =$ we can read 74 as 7*tens* and 4*ones* or 6*tens* and *14ones.* This is called **decomposing** numbers.

6 14



$$\begin{array}{r} \cancel{74} \\ -36 \\ \hline 38 \\ \hline \end{array}$$

74 so first we cannot take 6 from 4, so we decompose

now we can subtract 6 *ones* from 14*ones*

then we take 3 *tens* from 6 *tens*

Your turn

Use any method to solve the following:

- 632-258=
- 678-596=
- 325-58=

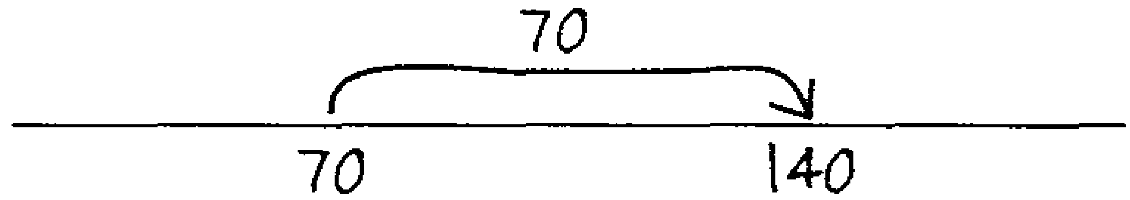
Your turn

- Use any method to solve the following:
- 632-258=374
- 678-596=83
- 325-58=267

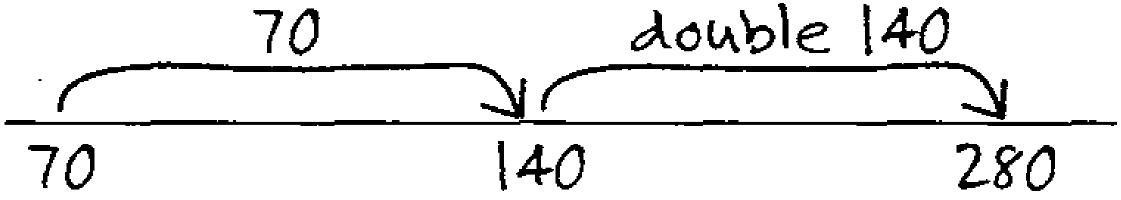
Multiplication

How much is 4 times 68?

I used 70s because they were easier than 68s.
First I did 70 and 70 is 140.



Then I doubled 140 to get 280.



Multiplication

Your turn with the number line

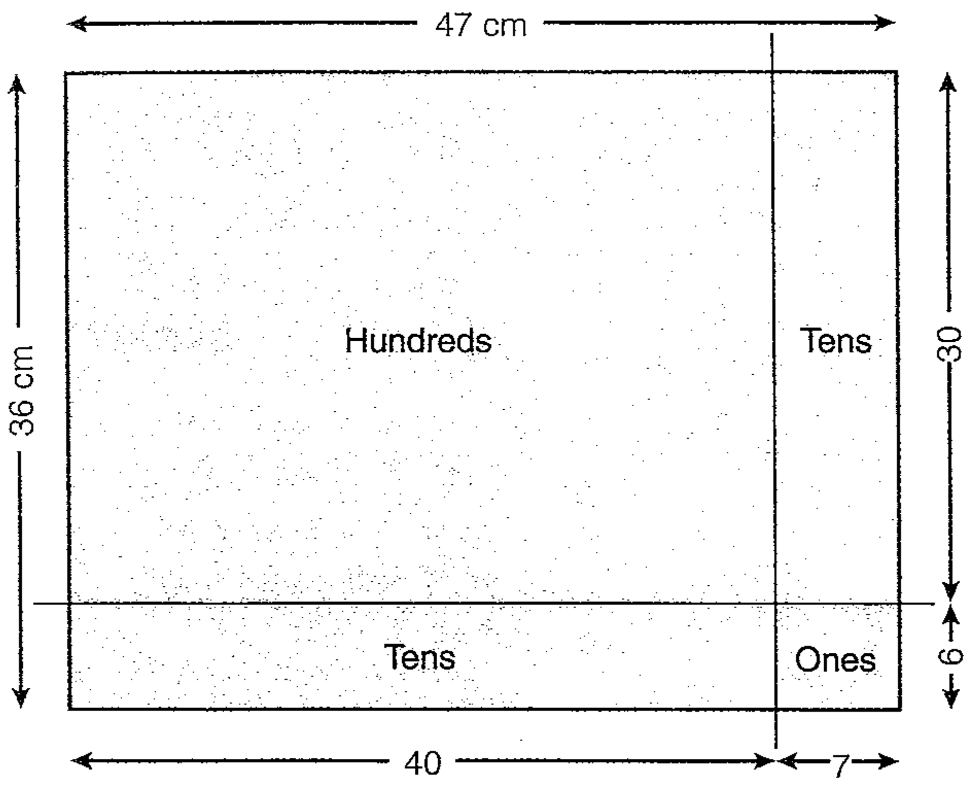
- 4x32
- 6x18

Traditional Algorithm explained

47
× 36

1200
210
240
42

1692



Multiplication

- Let’s look at 47×65
- Let’s estimate first $50 \times 60 = 3000$

4 **3**
47 multiply the **ones**, 7×5=35 rename 3tens and 5ones
× 65 5 × 4 is 20tens, add 3tens = 23 tens,
235 rename: 2hundreds 3tens and five ones
2820 multiply the **tens,** 6tens × 7ones = 42tens, **4** hundreds & 2tens
3055 6 tens × 4 tens are 24 hundreds plus 4 hundreds = 28
then add 235+2820=3055

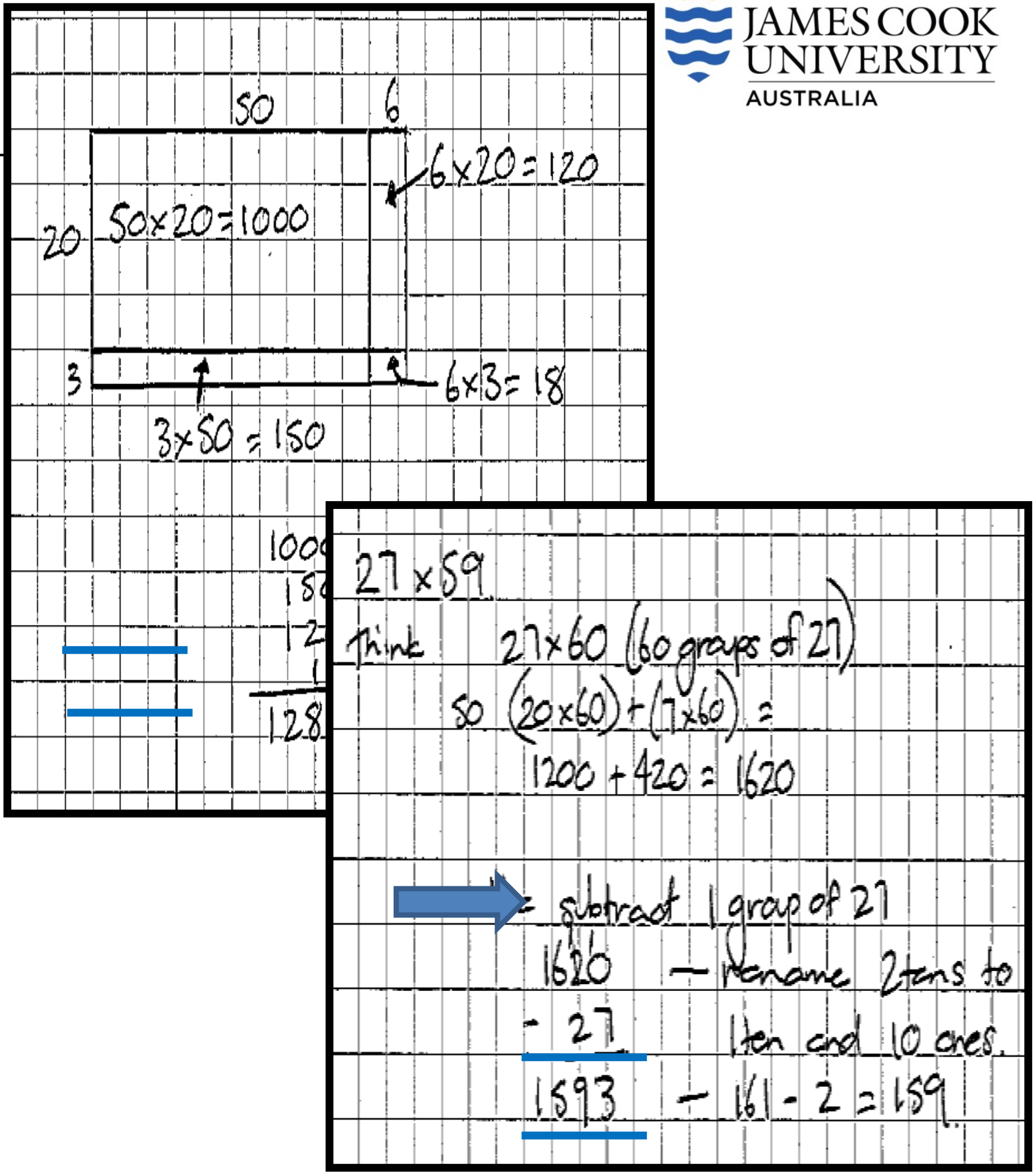
Your turn

Your turn:

- 89x100
- 56x23
- 27x59

Your turn

- Your turn:
- 89x100= 8900
- 56x23 = 1288
- 27x59 = 1593


50
6
20
50x20=1000
6x20=120
3
6x3=18
3x50=150

100
15
12
1
128

27 x 59

Think 27x60 (60 groups of 27)

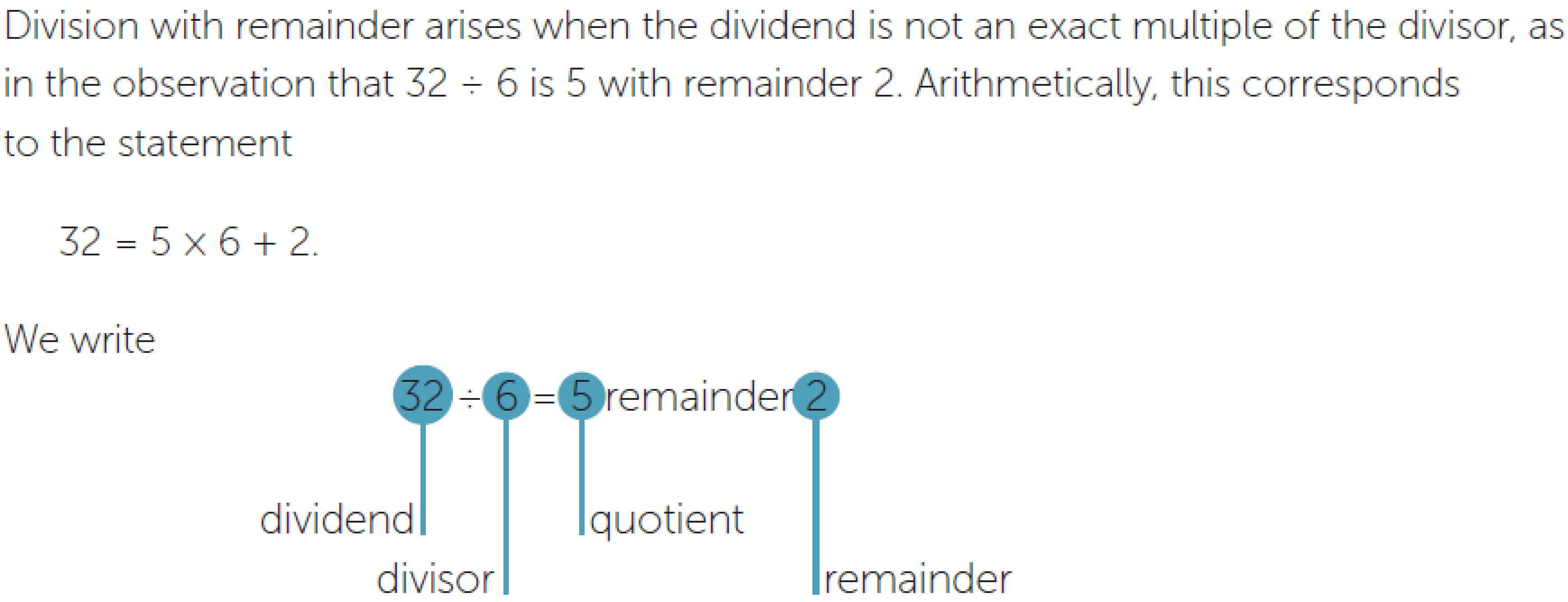
so (20x60)+(7x60) =

1200 + 420 = 1620

= subtract 1 group of 27

1620 — rename 2 tens to
\- 27 1 ten and 10 ones.
1593 — 161 - 2 = 159.

Division revision



Division with remainder arises when the dividend is not an exact multiple of the divisor, as in the observation that 32 ÷ 6 is 5 with remainder 2. Arithmetically, this corresponds to the statement

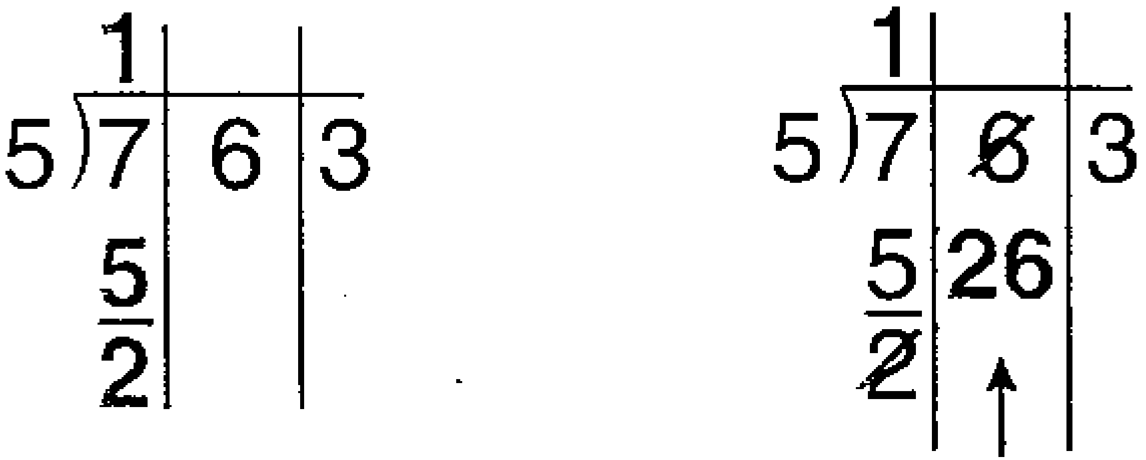
$$32 = 5 \times 6 + 2.$$

We write

$$\underbrace{32}_{\text{dividend}} \div \underbrace{6}_{\text{divisor}} = \underbrace{5}_{\text{quotient}} \text{ remainder } \underbrace{2}_{\text{remainder}}$$

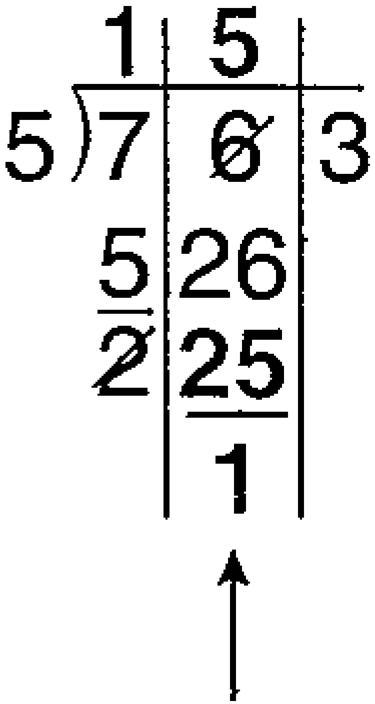
http://www.amsi.org.au/teacher_modules/pdfs/Whole_number_arithmetic.pdf

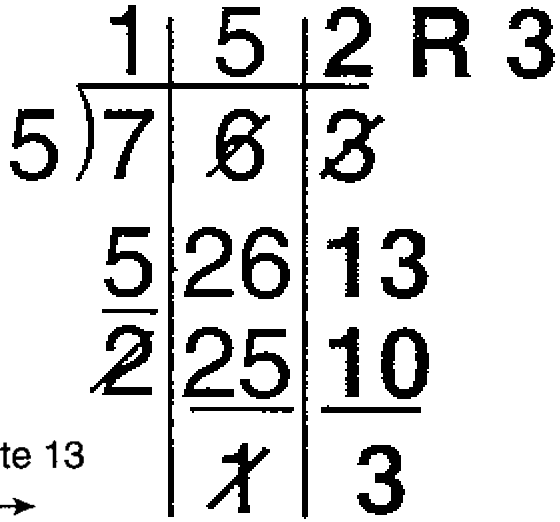
Division … Steps 1&2



Cross out the 2 and the 6. Write 26 in tens column.

Division…. Steps 3&4



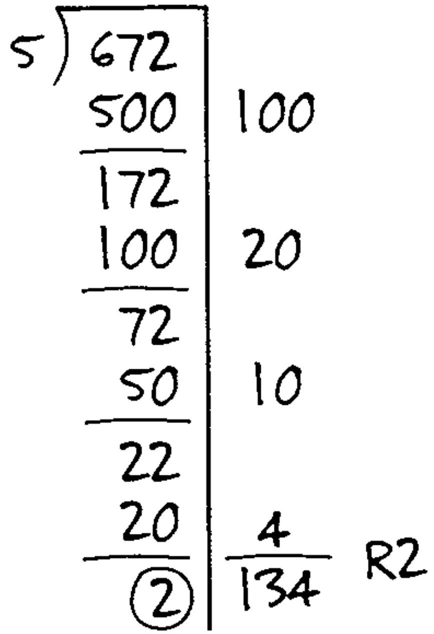


Cross out the 1 and the 3 and write 13 in the ones column. →

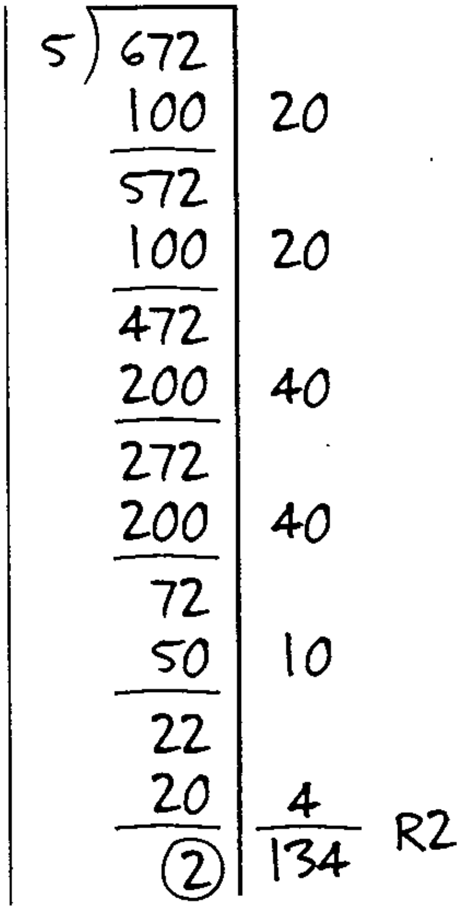
5 sets of 5 each is 5 × 5 = 25 tens.
Record the 25.
(Note two different ways of recording.) —
26 – 25 = 1 tells how many tens are left.

Division

- A new way…



5) 672	
500	100
172	
100	20
72	
50	10
22	
20	4
②	134 R2

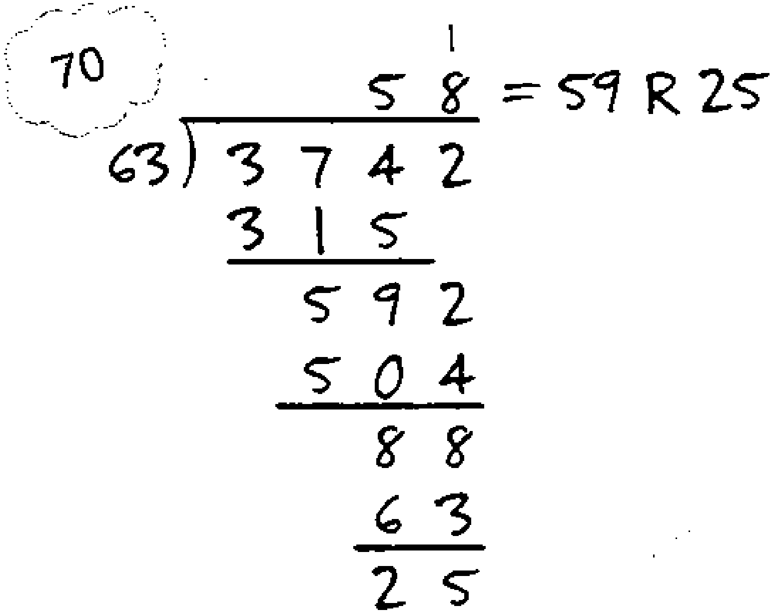


5) 672	
100	20
572	
100	20
472	
200	40
272	
200	40
72	
50	10
22	
20	4
②	134 R2

Division revision

- The old way

Traditional bring-down method

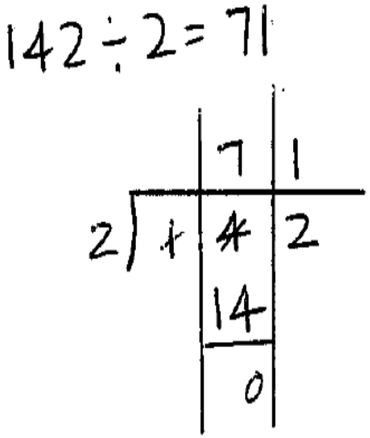


Your turn

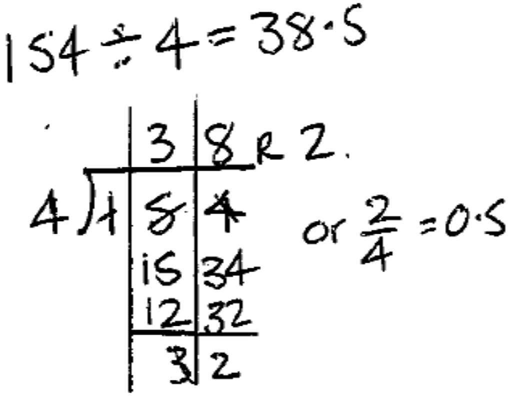
- $142 \div 2 =$
- $154 \div 4 =$
- $693 \div 9 =$
- $590 \div 25 =$
- $786 \div 15 =$

Your turn

- 142÷ 2 =71
- 154 ÷ 4 = 38.5
- 693 ÷ 9 =77
- 590 ÷ 25 =23.6
- 786 ÷ 15 =52.4



142÷2=71



154÷4=38·5

Examples….

- $590 \div 25 = 23{\cdot}6$

25	590	
	100	4
	490	
	400	16 (4×4)
	90	
	50	2
	40	
	25	1
	15	R

$4+16+2+1 = 23\,^{15}/_{25}$

$= 23\,^{3}/_{5}$

$= 23\,^{6}/_{10}$

$= 23{\cdot}6$

$786 \div 15 = 52{\cdot}4$

15	786	
	150	10
	636	
	300	20
	336	
	300	20
	36	
	30	2
	6	R

$10+20+20+2$

$= 52\,^{6}/_{15}$

$= 52\,\frac{2}{5}$

Working with Decimals

Reflect on the intent of this workshop….

- The decimal separates whole numbers from parts of a whole.
- Each digit in a number has a ‘place value’
- The value depends on the position of the digit in that number
- Each position can be thought of as columns
- Each column is a power of ten.