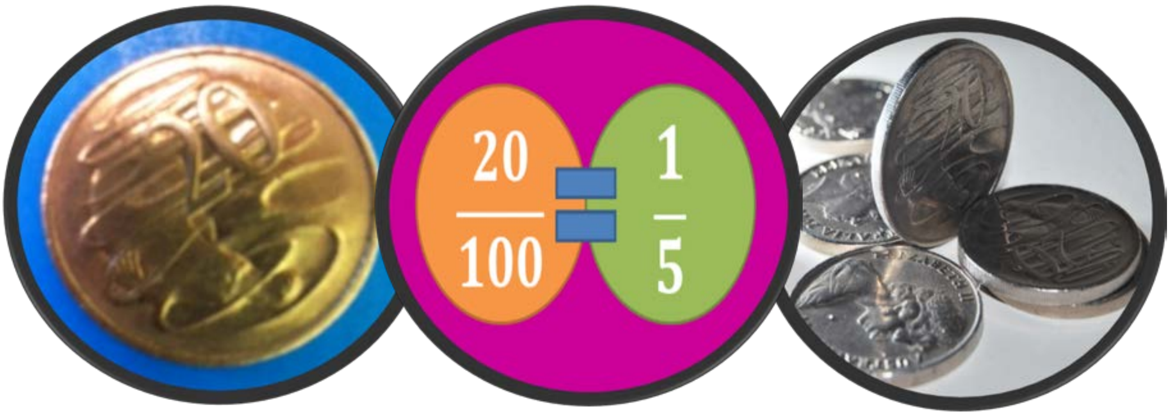
Maths Module 2

Working with Fractions

This module covers concepts such as:

- identifying different types of fractions
- converting fractions
- addition and subtraction of fractions
- multiplication and division of fractions



www.jcu.edu.au/students/learning-centre

TheLearningCentre
UNLOCK YOUR POTENTIAL

Module 2

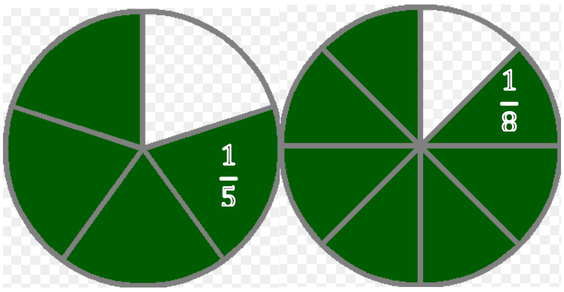
Fractions

1. Fraction Glossary

Fractions are representations of ‘even parts of a whole.’ Fraction parts are equal-sized shares (or portion) of ***one*** whole. The whole can be **one object** or a **collection of things:** ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ . Hence, the collection of ten smiling faces can be the whole; the collective group is a whole. Five of them would be half of the whole group of 10. Let’s relate to money, if $50 made up the whole, then $10 would be one fifth of the whole ($50).

More abstractly the unit (whole) is counted as 1. On the number line, the distance from 0-1 is the unit. Fractions can be represented as: common ($\frac{3}{4}$); decimal (0.75); or, as a percent (75%).
Fraction parts have special names that tell how many parts of that size are needed to make the whole. For example: thirds $\left(\frac{1}{3}\right)$ require 3 equal parts to make the whole, three thirds $\frac{3}{3} = 1$.

The more parts in the whole, the smaller the parts. As demonstrated below, an eighth ($\frac{1}{8}$) is smaller than a fifth ($\frac{1}{5}$); when the circle is divided into eighths, there are more parts than fifths, and they are smaller parts.



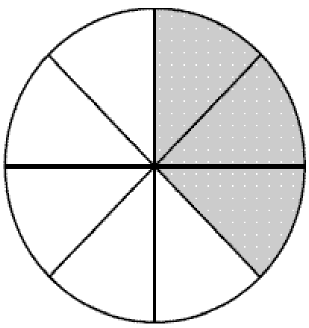
Equivalent fractions are two ways of describing the same amount by using different sized fractional parts.
A key concept is that division and fractions are linked. An easy way to remember is that the ‘line’ (vinculum) in a fraction means to divide by. Even the division symbol (÷) is a fraction. Also, remember that division and multiplication are linked; division is the opposite of multiplication. For instance, $2 \times 3 = 6$ means 6 is two groups of three, and thus $6 \div 2 = 3$, which means that six can be divided into two groups of three.

A fraction is made up of two main parts: $\frac{3}{4} \rightarrow \frac{Numerator}{Denominator}$

Denominator represents how many even parts of the whole there are, and the **numerator** signifies how many of those even parts are of interest.

$$\frac{5}{8}\ of\ a\ pizza\ means\ you\ have\ cut\ the\ pizza\ into\ 8\ even\ pieces\ and\ you\ have\ 5\ of\ them.$$

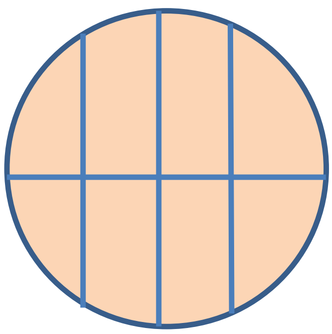
A.



1. Your Turn:

Why is B. not an accurate representation of $\frac{8}{8}$?

B.



2. Naming Fractions

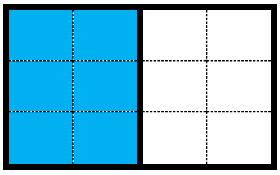
Fractions should always be displayed in their *simplest* form. For example, $\frac{6}{12}$ is converted to $\frac{1}{2}$

Let's investigate how this works:

What information is the fraction providing us? The denominator says that we have 12 equal parts and the numerator says that we are dealing with 6 or those 12. It makes it easier to visualise this, as below:

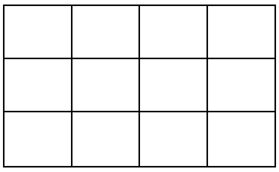
1	4	7	10
2	5	8	11
3	6	9	12

Now let's shade 6 of those 12 equal parts



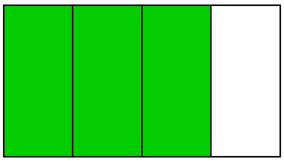
What other relationships can we see now?

We can see 2 equal parts, one is blue and the other is white, each one of those is equivalent to 6 parts of 12 equal parts $\frac{6}{12}$ *or* $\frac{1}{2}$



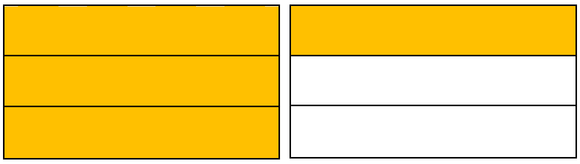
Use the blank rectangle to shade other fractions – for example thirds or quarters

- A **proper fraction** has a numerator smaller than the denominator, for example: $\frac{3}{4}$



This representation shows that we have four equal parts and have shaded three of them, therefore: $\frac{3}{4}$

- An **improper fraction** has a numerator larger than the denominator, for instance: $\frac{4}{3}$



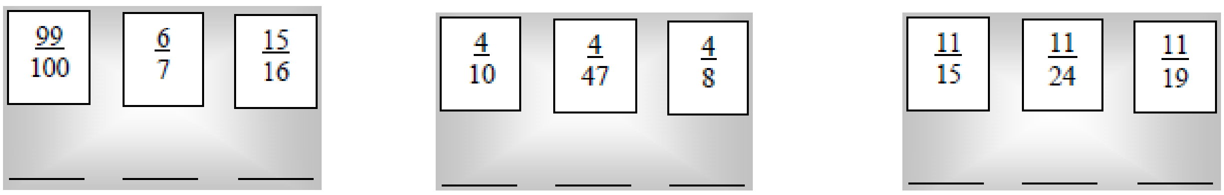
Here we have two 'wholes' divided into three equal parts.

Three parts of '3 equal parts' makes a 'whole' plus one more part makes '*one whole and one third*' or '*four thirds*'

- Therefore, a **mixed fraction** has a whole number and a fraction, such as: $1\frac{1}{3}$

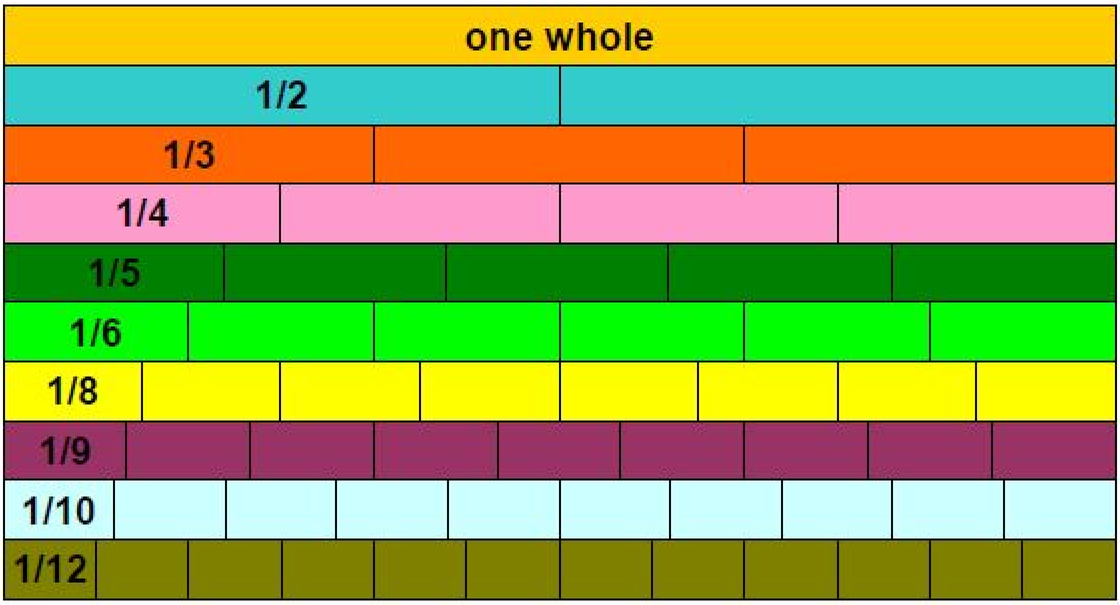
2. Your Turn:

Reflect on the idea that each fraction represents a 'fraction of one whole unit', and then have a go at ordering these fractions (use the number line to assist your thinking and reasoning).



3. Equivalent Fractions

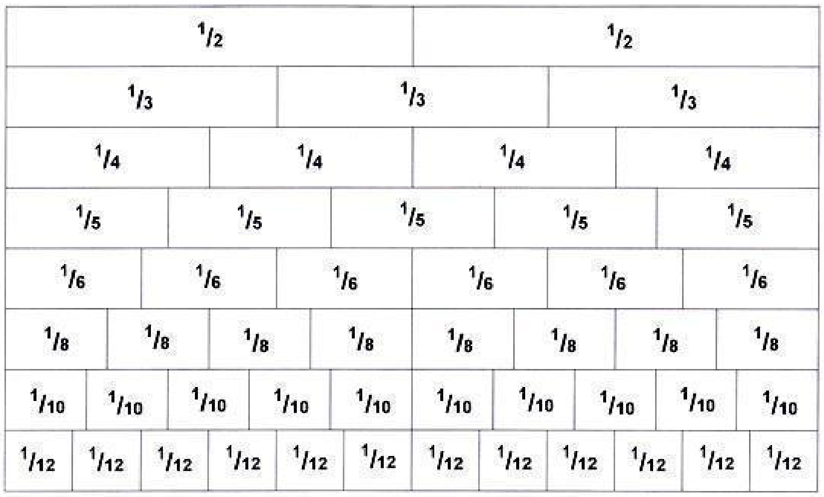
Equivalence is a concept that can be easy to understand when a fraction wall is used.



As you can see, each row has been split into different fractions; the top row is one (whole) and then the next is divided into 2 (halves), the bottom row has been divided into 12 (twelfths). An equivalent fraction splits the row at the same place. This helps us to reason why 'the greater the denominator the smaller the part'.

Therefore:

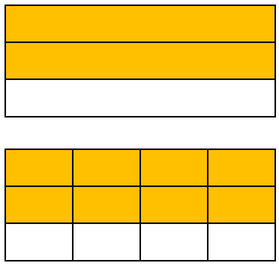
$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}$ To prove this idea to yourself, shade the fractions below to correspond.



The more pieces I split the row into (denominator), the more pieces I will need (numerator) to create an equivalent fraction. Mathematically, whatever I do to the denominator (multiply or divide), I must also do to the numerator and vice versa.

Take $\frac{2}{3}$ as an example; look at it on the fraction wall and then convert to twelfths. To do this abstractly, we multiply the numerator by 4. We do this because we look at the denominator and see what we have done to the digit 3 (thirds) to make it 12 (twelfths). We have multiplied the denominator by 4. So to convert the fraction we must also multiply the numerator by 4: $\frac{2\times4}{3\times4}=\frac{8}{12}$

Let's investigate with an image



Take a rectangle and divide it into three equal parts and shade 2, to get $\frac{2}{3}$

Now to convert to twelfths, we divide the same rectangle (the whole) into 12 equal parts. We can see that we have divided each of the thirds by 4. So now if we count up the parts, we have 8 parts of 12 shaded.

$\therefore\ \frac{2\times 4}{3\times 4} = \frac{8}{12}$ meaning $\frac{2}{3} = \frac{8}{12}$

EXAMPLE PROBLEMS:

1. $\frac{3}{5} = \frac{\quad}{20}$ Answer: The denominator was multiplied by 4. (20 ÷5 =4)

   So the numerator must be multiplied by 4. $\therefore\ \frac{3^{\times 4}}{5_{\times 4}} = \frac{12}{20}$

2. $\frac{27}{81} = \frac{9}{\quad}$ Answer: The numerator was divided by 3. (27 ÷9 =3)

   So the denominator must be divided by 3. $\therefore\ \frac{27^{\div 3}}{81_{\div 3}} = \frac{9}{27}$

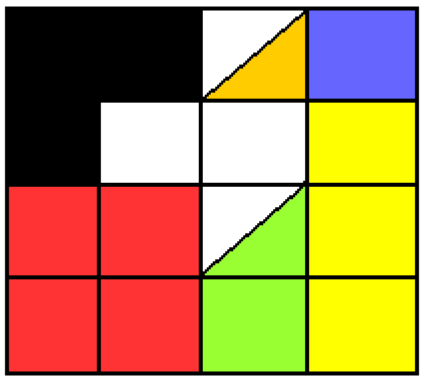
3. Your Turn:

a. $\frac{2}{3} = \frac{\quad}{9}$

b. $\frac{5}{7} = \frac{45}{\quad}$

c. $\frac{9}{10} = \frac{\quad}{30}$

d. $\frac{\quad}{52} = \frac{4}{13}$



http://www.analyzemath.com/primary_math/grade_5/fractions_sol.html

e. What fraction of the large square is red?

f. What fraction of the large square is blue?

g. What fraction of the large square is orange?

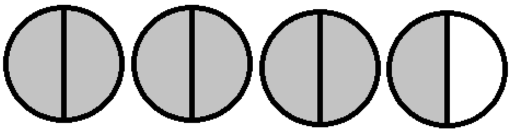
h. What fraction of the large square is green?

i. What fraction of the large square is black?

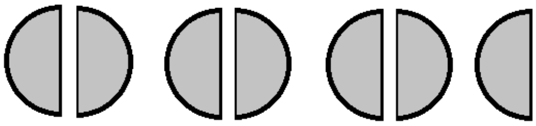
j. What fraction of the large square is yellow?

4. Converting Mixed Numbers to Improper Fractions

A mixed number is a way of expressing quantities greater than 1. A mixed number represents the number of wholes and remaining parts of a whole that you have, while an improper fraction represents how many parts you have. The diagram below illustrates the difference between a mixed number and an improper fraction, using a quantity of car oil as an example. On the left, we use a mixed number to represent 3 whole litres and 1 half litre. We write this mixed number as 3 ½. On the right, we use an improper fraction to represent 7 half litres. We write this improper fraction as $\frac{7}{2}$.



Is the same as



$$3\frac{1}{2} = 7\ halves = \frac{7}{2}$$

You are more likely to encounter mixed numbers than improper fractions in everyday language. For example, you might say, 'my car requires 3 ½ litres of oil,' rather than, 'my car requires $\frac{7}{2}$ litres of oil.'

It is much easier to multiply or divide fractions when they are in improper form. As such, mixed numbers are usually converted to improper fractions before they are used in calculations. To convert from a mixed number to an improper fraction, multiply the whole number by the denominator then add the numerator. This total then becomes the new numerator which is placed over the original denominator. For example:

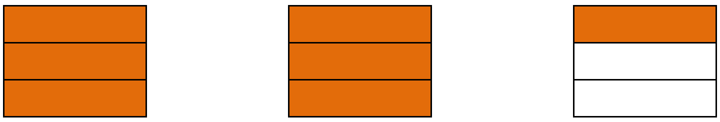
Convert $3\frac{1}{2}$ into an improper fraction.

Working: $3(whole\ number) \times 2(denominator) + 1(numerator) = 7$

Therefore, $the\ improper\ fraction\ is\ \frac{7}{2}$

EXAMPLE PROBLEMS:

1. $2\frac{2}{3} = \frac{8}{3}$ $Note$: $(2 \times 3 + 2 = 8)$ which is (8 lots of $\frac{1}{3}$)



2. $2\frac{3}{7} = \frac{17}{7}$ $Note$: $(2 \times 7 + 3 = 17)$ which is (17 lots of $\frac{1}{7}$)

4. Your Turn:

a. $4\frac{1}{2} = -$

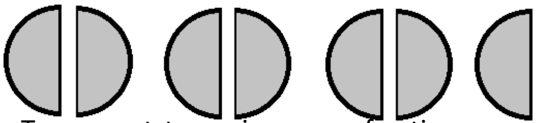
b. $5\frac{1}{3} = -$

c. $7\frac{3}{5} = -$

d. $2\frac{1}{8} = -$

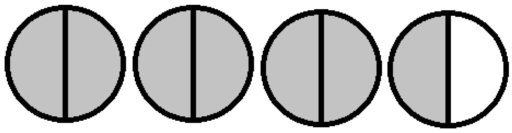
5. Converting Improper Fractions to Mixed Numbers

While improper fractions are good for calculations, they are rarely used in everyday situations. For example, people do not wear a size $\frac{23}{2}$ shoe; instead they wear a size $11\frac{1}{2}$ shoe.



$$= seven\ halves = \frac{7}{2}$$

To convert to an improper fraction we need to work out how many whole numbers we have. Here we reverse the procedure from the previous section. We can see that 6 of the halves combine to form 3 wholes; with a half left over.



$$= 3\frac{1}{2}$$

So to work this symbolically as a mathematical calculation we simply divide the numerator by the denominator. Whatever the remainder is becomes the new numerator. Using a worked example of the diagram above: Convert $\frac{7}{2}$ into a mixed fraction:

$7 \div 2 = 3\frac{1}{2}$ If I have three whole numbers, then I also have six halves and we have one half remaining. ∴ $\frac{7}{2} = 3\frac{1}{2}$

That was an easy one. Another example:

Convert $\frac{17}{5}$ into a mixed fraction:

Working: $17 \div 5 =$ the whole number is 3 with some remaining.

If I have 3 whole numbers, that is 15 $fifths$. (3×5)

Then I must now have 2 $fifths$ remaining. $(17 - 15)$

$\therefore 3\frac{2}{5}$

EXAMPLE PROBLEMS:

1. $\frac{27}{6} = 4\frac{3}{6} = 4\frac{1}{2}$
   $(4 \times 6 = 24)\ (27 - 24 = 3)$
   also remember equivalent fractions.
   Note: $(27 \div 6 = 4.5)$

2. $\frac{8}{3} = 2\frac{2}{3}$ $(2 \times 3 = 6)\ (8 - 6 = 2)$
   Note:(8÷3=2.67)

5. Your Turn:

a. $\frac{7}{5} =$

c. $\frac{53}{9} =$

b. $\frac{27}{7} =$

d. $\frac{12}{9} =$

6. Converting Decimals into Fractions

Decimals are a universal method of displaying data, particularly given that it is much easier to enter decimals rather than fractions into computers. But fractions can be more accurate, e.g. $\frac{1}{3}$ is not 0.33 it is $0.3\dot{3}$

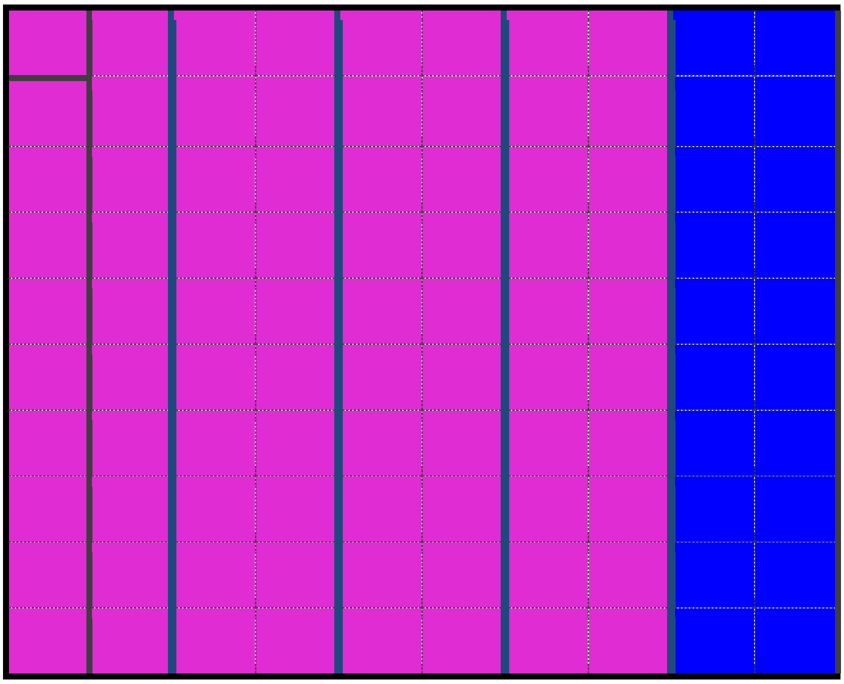
The method used to convert decimals into fractions is based on the notion of place value (see Module One for more details). The place value of the <u>last</u> digit in the decimal determines the denominator, tenths, hundredths, thousandths and so on: for instance $0.3 = \frac{3}{10}$ and $0.63 = \frac{63}{100}$

Units (whole number)	Tenths	Hundredths	Thousandths

The decimal separates the whole number from parts of a whole.

EXAMPLES:

1. 0.8 has 8 in the tenths column, yet if we add a zero to read 0.80 we can say we have 80 parts of 100 Therefore, $\frac{80}{100}$ or 0.80 is equivalent to 0.8 or $\frac{8}{10} = \frac{4}{5}$ (remember equivalent fractions!)



The blue section represents $\frac{20}{100}$ or $\frac{2}{10}$ or $\frac{1}{5}$

2. 0.5 is 5 tenths. Thus 0.5 is $\frac{5}{10} = \frac{1}{2}$ (converted to a simpler, equivalent fraction). Mark this on the grid as a way to understand the relationship conceptually.
3. 0.375 is 375 thousandths. Let' convert $\frac{375}{1000}$ to a simpler fraction: $\frac{375^{\div 125}}{1000_{\div 125}} = \frac{3}{8}$
4. 1.25 is 125 hundredths, so we have a whole number $\frac{100}{100}$ and a fraction of a whole $\frac{25}{100}$ Hence, we have 1 and $\frac{25^{\div 25}}{100_{\div 25}} = 1\frac{1}{4}$

The hardest part is converting to the lowest equivalent fraction. Thankfully if you have a scientific calculator you can use the fraction button.

Enter 375 then  followed by 1000 press = and answer shows as 3⌟8 = $\frac{3}{8}$.

NOTE: Calculator does not work for rounded decimals, especially thirds. For instance, $0.333 \approx \frac{1}{3}$

The table on the following page lists some commonly encountered fractions expressed in their decimal form:

Decimal	Fraction
0.125	$\frac{1}{8}$
0.25	$\frac{1}{4}$
0.33333	$\frac{1}{3}$
0.375	$\frac{3}{8}$
0.5	$\frac{1}{2}$
0.66667	$\frac{2}{3}$
.75	$\frac{3}{4}$

6. Your Turn: (No Calculator first, then check!)

a. 0.65 =

b. 2.666 =

c. 0.54 =

d. 3.14 =

7. Converting Fractions into Decimals

Converting fractions into decimals is based on place value. For example, applying what you have understood about equivalent fractions, we can easily convert $\frac{2}{5}$ into a decimal. First we need to convert to a denominator that has a 10 base. Let's convert $\frac{2}{5}$ into tenths $\rightarrow \frac{2^{\times 2}}{5_{\times 2}} = \frac{4}{10}$ ∴ we can say that two fifths is the same as four tenths: 0.4

Converting a fraction to decimal form is a simple procedure because we simply use the divide key on the calculator. Note: If you have a mixed number, convert it to an improper fraction before dividing it on your calculator.

Example Problems:

1. $\frac{2}{3} = 2 \div 3 = 0.66666666666etc \ \approx 0.67$
2. $\frac{3}{8} = 3 \div 8 = 0.375$
3. $\frac{17}{3} = 17 \div 3 = 5.6666666etc \ \approx 5.67$
4. $3\frac{5}{9} = 3 + 5 \div 9 = 3 + 0.5555555etc \ \approx 3.56$ (remember the BODMAS rule)

7. Your Turn: (round your answer to three decimal places where appropriate)

a. $\frac{17}{23} =$

b. $\frac{5}{72} =$

c. $56\frac{2}{3} =$

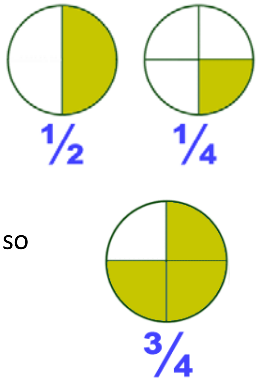
d. $\frac{29}{5} =$

8. Fraction Addition and Subtraction

Adding and subtracting fractions draws on the concept of equivalent fractions. The golden rule is that you can only **add** and **subtract** fractions if they have the **same denominator**. For example, $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$.

However, if two fractions do ***not*** have the same denominator, we must use equivalent fractions to find a “common denominator” before they can be added together or subtracted.

An analogy is if we were to add measurements, they need to be in the same unit or else it will not make sense. For example, if we had 1m and we were required to add 16cm we can’t simply add 1+16 to get 17. What would the 17 refer to, metres or centimetres? We need to know what size each of the measurements are, and then add accordingly. Thus we would convert to a common unit, such as centimetres. So, 1m now becomes 100cm, and then we add to this the 16cm to make 116cm. The same mathematical thinking applies when working with addition or subtraction of fractions.



EXAMPLES: $\frac{1}{4}+\frac{1}{2}$ have different denominators, so why is this not true: $\frac{1}{4}+\frac{1}{2}=\frac{2}{6}$

We can see that $\frac{2}{6}$ *is equivalent to* $\frac{1}{3}$; not a sensible result for $\frac{1}{4}+\frac{1}{2}$

Instead we ‘convert’ the fractions to have the **same denominator**:

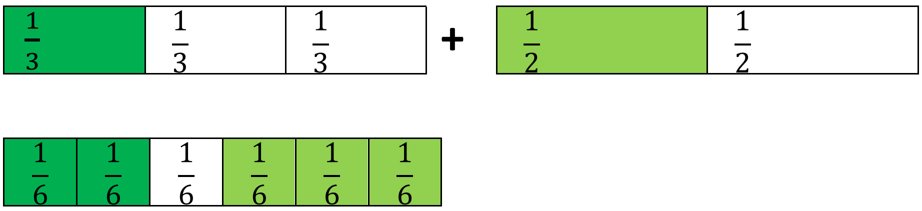
$\frac{1}{2}$ can be converted to $\frac{2}{4}$ Now the fractions have the same denominator and so we can carry out the addition: $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$

Use graphic representations to help make sense of the concept.

EXAMPLE 2: $\frac{1}{3}+\frac{1}{2}=?$

We cannot simply add these fractions; again we need to find the **lowest common denominator**. The easiest way to do this is to multiply the denominators: $\frac{1}{3}$ *and* $\frac{1}{2}$ (2 x 3 = 6). Therefore, both fractions can have a denominator of 6, yet we need to change the **numerator**. The next step is to convert both fractions into sixths as an equivalent form. How many sixths is one third? $\frac{1}{3}=\frac{2}{6}$ $\left(\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}\right)$ And how many sixths is one half? $\frac{1}{2}=\frac{3}{6}$ $\left(\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}\right)$

Therefore: $\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$



The abstract working: $\frac{1^{\times 2}}{3_{\times 2}}+\frac{1^{\times 3}}{2_{\times 3}}=\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$

KEY IDEA: WHAT WE DO TO ONE SIDE WE MUST DO TO THE OTHER.

With practise a pattern forms: $\frac{1}{3}+\frac{2}{5}=\frac{(1\times5)+(2\times3)}{(3\times5)}=\frac{5+6}{15}=\frac{11}{15}$

In the example above, the lowest common denominator is found by multiplying 3 and 5, and then the numerators are multiplied by 5 and 3 respectively.

Use the following problems to reinforce the pattern:

EXAMPLE ADDITION PROBLEMS:

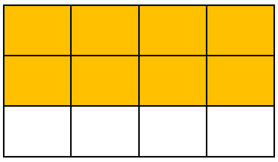
1. $\frac{3}{4}+\frac{2}{7}=\frac{(3\times7)+(2\times4)}{(4\times7)}=\frac{21+8}{28}=\frac{29}{28}=1\frac{1}{28}$

2. $\frac{5}{9}+\frac{3}{7}=\frac{(5\times7)+(3\times9)}{(9\times7)}=\frac{35+27}{63}=\frac{62}{63}$

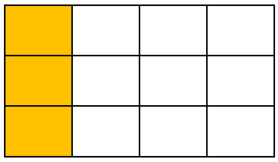
3. $2\frac{2}{3}+1\frac{3}{4}=\frac{8}{3}+\frac{7}{4}=\frac{(8\times4)+(7\times3)}{(3\times4)}=\frac{32+21}{12}=\frac{53}{12}=4\frac{5}{12}$ Note: First convert mixed to improper fractions and then convert back to a mixed fraction.

Subtraction follows the same procedure with a negative symbol:

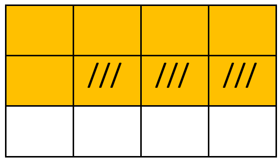
$$\frac{2}{3}-\frac{1}{4}=\frac{(2\times4)-(1\times3)}{(3\times4)}=\frac{8-3}{12}=\frac{5}{12}$$



$\frac{2}{3}$ *or* $\frac{8}{12}$



$\frac{1}{4}$ *or* $\frac{3}{12}$



$\frac{5}{12}$

EXAMPLE SUBTRACTION PROBLEMS:

1. $\frac{9}{12}-\frac{1}{3}=\frac{(9\times3)-(1\times12)}{(12\times3)}=\frac{27-12}{36}=\frac{15^{\div3}}{36_{\div3}}=\frac{5}{12}$ Converting to the simplest form

2. $\frac{1}{3}-\frac{1}{2}=\frac{(1\times2)-(1\times3)}{(3\times2)}=\frac{2-3}{6}=\frac{-1}{6}=-\frac{1}{6}$ Note: you have taken more then you originally had.

3. $3\frac{2}{7}-\frac{9}{12}=\frac{23}{7}-\frac{9}{12}=\frac{(23\times12)-(9\times7)}{(7\times12)}$

   $=\frac{276-63}{84}=\frac{213}{84}$ Now we need to convert the improper ration to a mixed number

   $=2\frac{45}{84}$ Divide 213 by 84 to get 2 with 45 remaining

   $=2\frac{15}{28}$ Convert to simplest form, numerator and denominator ÷3.

8. Your Turn:

a. $\frac{1}{3}+\frac{2}{5}=$

c. $\frac{5}{6}-\frac{1}{4}=$

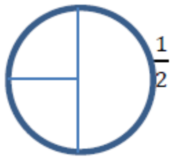
b. $2\frac{1}{6}+3\frac{7}{8}=$

d. $3\frac{12}{23}-2\frac{4}{7}=$

Use lots of space for working and to show your thinking and reasoning, the more you practise the more it will make sense to you….

9. Fraction Multiplication and Division

Compared to addition and subtraction, multiplication and division of fractions is easy to do, but sometimes a challenge to understand how and why the procedure works mathematically. For example, imagine I have $\frac{1}{2}$ of a pie and I want to share it between 2 people. Each person gets a quarter of the pie.



$\frac{1}{2} \, of \, \frac{1}{2} = \frac{1}{4}$

Mathematically, this example would be written as: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Remember that fractions and division are related; in this way, multiplying by $\frac{1}{2}$ is the same as dividing by two.

Hence, $\frac{1}{2}$ (two people to share) of $\frac{1}{2}$ (the amount of pie) is $\frac{1}{4}$ (the amount each person will get).

But what if the question was more challenging: $\frac{2}{3} \times \frac{7}{16} =$? This problem is not as easy as splitting pies.

A mathematical strategy to use is: “Multiply the numerators then multiply the denominators”

Therefore, $\frac{2}{3} \times \frac{7}{16} = \frac{(2\times7)}{(3\times16)} = \frac{14}{48} = \frac{7}{24}$

However, we can also apply a cancel out method – which you may recall from school. The rule you may recall is, ‘What we do to one side, we must do to the other.’

Thus, in the above example, we could simplify first: $\frac{2}{3} \times \frac{7}{16} =$? The first thing we do is look to see if there are any common multiples. Here we can see that 2 is a multiple of 16, which means that we can divide top and bottom by 2: $\frac{2^{\div 2}}{3} \times \frac{7}{16_{\div 2}} = \frac{1}{3} \times \frac{7}{8} = \frac{1\times7}{3\times8} = \frac{7}{24}$

Example Multiplication Problems:

1. $\frac{4}{9} \times \frac{3}{4} = \frac{(4\times3)}{(9\times4)}$

   $= \frac{12}{36}$

   $= \frac{1}{3}$

   Another way to approach this is to apply a cancelling out method.

   $\frac{4}{9} \times \frac{3}{4} = \frac{(4^{\div 4}\times3)}{(9\times4_{\div 4})}$ At this step we could divide both the numerator and the denominator by 4

   $= \frac{3}{9}$ Then we simplify further by dividing both by 3 to get $\frac{1}{3}$

2. $2\frac{4}{9} \times 3\frac{3}{5} = \frac{22}{9} \times \frac{18}{5}$

   $= \frac{(18\times22)}{(9\times5)}$

   $= \frac{396}{45} = \frac{44}{5}$

   Or we could apply the cancelling method $\frac{(18^{\div 9}\times22)}{(9_{\div 9}\times5)} = \frac{2\times22}{1\times5} = \frac{44}{5}$

   $= 8\frac{4}{5}$

9. Your Turn:

a. $\frac{5}{8} \times 40 =$

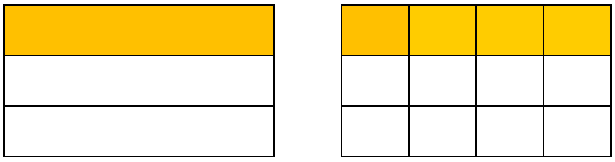
b. $\frac{5}{9} \times 63 =$

c. $\frac{2}{3} \times \frac{7}{13} =$

d. $1\frac{1}{6} \times \frac{2}{9} =$

Division of Fractions

The same process applies below when we divide one third into four equal parts: $\frac{1}{3} \div 4 = ?$



The rectangle (the whole) is first divided into three equal parts to represent 'thirds'. Then on the second rectangle the thirds have then been divided into 4 equal parts and we can see that we now have 12 parts, each being $\frac{1}{12}$

Therefore, $\frac{1}{3} \div \frac{4}{1}$ is worked mathematically as $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

As the representation shows, when dividing one third into four equal pieces, each of piece will represent $\frac{1}{12}$

Therefore, $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ 'a third *times* a quarter' is the same as 'a third *divided* into four'

Division might seem odd, but is a simple concept. $\div \frac{1}{2}$ *is the same as* $\times \frac{2}{1}$ and $\div 4$ *is the same as* $\times \frac{1}{4}$

Now let's look at: 'a third divided by a quarter' is the same as 'a third *times* four'

$\frac{1}{3} \div \frac{1}{4} = ?$ *is the same thing as* $\frac{1}{3} \times \frac{4}{1} = ?$

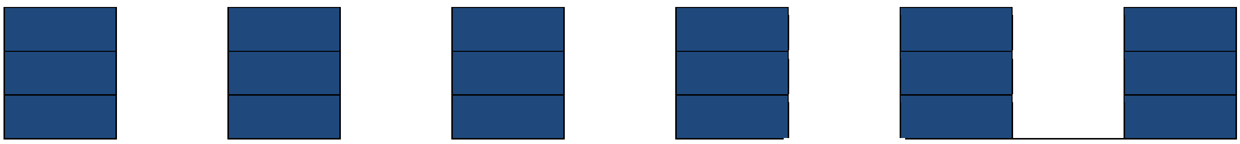
This is harder to comprehend but if we follow our rules that when we divide, we flip the fraction, then we have a third times four. Therefore, $\frac{1}{3} \times \frac{4}{1} = \frac{4}{3}$ *or* $1\frac{1}{3}$

If the sign is swapped to its opposite, the fraction is flipped upside down, which is referred to as the ***reciprocal of the original fraction***.

Therefore, $\frac{2}{3} \div \frac{1}{2}$ *is the same as* $\frac{2}{3} \times \frac{2}{1} = \frac{(2\times2)}{(3\times1)} = \frac{4}{3} = 1\frac{1}{3}$ Note: dividing by half doubled the answer.

So as to not confuse yourself, remember that multiplication and division are linked and that we must apply the reciprocal rule for division. You may recall the rule from school as 'invert and multiply'.

HERE IS AN EXAMPLE TO DEMONSTRATE THE RULE: $6 \div \frac{1}{3} = ?$ 6 whole units divided into thirds: we have 18 thirds



or

Or if we think of division as repeated subtraction, then we repeatedly subtract $\frac{1}{3}$ from 6. 18 pieces can be subtracted, each piece being $\frac{1}{3}$

EXAMPLE DIVISION PROBLEMS:

1. $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3}$
$= \frac{(2\times5)}{(3\times3)}$
$= \frac{10}{9}\ simplified = 1\frac{1}{9}$

2. $3\frac{3}{4} \div 2\frac{2}{3} = \frac{15}{4} \div \frac{8}{3}$
$= \frac{15}{4} \times \frac{3}{8}$
$= \frac{(15\times3)}{(4\times8)}$
$= \frac{45}{32}\ simplified = 1\frac{13}{32}$

9. Your Turn:

e. $15 \div \frac{2}{5} =$

f. $\frac{3}{7} \div \frac{8}{21} =$

g. $\frac{3}{7} \div \frac{2}{5} =$

h. $2\frac{2}{5} \div 3\frac{8}{9} =$

10. Answers

1)	The parts must be even parts.					
2)	$\frac{6}{7}, \frac{15}{16}, \frac{99}{100};$	$\frac{4}{47}, \frac{4}{10}, \frac{4}{8};$	$\frac{11}{24}, \frac{11}{19}, \frac{11}{15}$			
3)	a. $\frac{6}{9}$	b. $\frac{45}{63}$	c. $\frac{27}{30}$	d. $\frac{16}{52}$		
	e. $\frac{1}{4}$	f. $\frac{1}{16}$	g. $\frac{1}{32}$	h. $\frac{3}{32}$	i. $\frac{3}{16}$	j. $\frac{3}{16}$
4)	a. $\frac{9}{2}$	b. $\frac{16}{3}$	c. $\frac{38}{5}$	d. $\frac{17}{8}$		
5)	a. $1\frac{2}{5}$	b. $3\frac{6}{7}$	c. $5\frac{8}{9}$	d. $1\frac{1}{3}$		
6)	a. $\frac{13}{20}$	b. $2\frac{2}{3}$	c. $\frac{27}{50}$	d. $3\frac{7}{50}$		
7)	a. 0.739	b. 0.069	c. 56.667	d. 5.8		
8)	a. $\frac{11}{15}$	b. $6\frac{1}{24}$	c. $\frac{7}{12}$	d. $\frac{153}{161}$		
9)	a. 25	b. 35	c. $\frac{14}{39}$	d. $\frac{7}{27}$		
	e. $37\frac{1}{2}$	f. $1\frac{1}{8}$	g. $1\frac{1}{14}$	h. $\frac{108}{175}$		

11. Helpful Websites

Equivalent Fractions: http://www.mathsisfun.com/equivalent_fractions.html

Mixed Fraction: http://www.jamit.com.au/htmlFolder/FRAC1003.html

Decimals: http://www.mathsisfun.com/converting-fractions-decimals.html

Addition Subtraction: http://www.mathsisfun.com/fractions_addition.html

Multiplication Division: http://www.mathsisfun.com/fractions_multiplication.html