Basic Statistics

Hypothesis Testing

Hypothesis Testing

Learning Intentions

Today we will understand:

- Formulating the null and alternative hypothesis
- Distinguish between a one-tail and two-tail hypothesis test
- Controlling the probability of a Type I and Type II error

Hypothesis

- A hypothesis is an assumption about a population parameter

Examples of hypotheses:

- The average adult drinks 1.8 cups of tea per day

- Thirteen percent of high school students in Australia will go straight to university

"I've narrowed it to two hypotheses: it grew or we shrunk."

Image accessed: http://www.truthandscience.net/thenatureofscience.htm

Hypothesis

- A statement about the population that may or may not be true
- Hypothesis testing aims to make a statistical conclusion about accepting or not accepting the hypothesis

Image accessed: http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.108.170502

Hypothesis

- The best way to determine if a hypothesis was true would be to examine the entire population
- Usually impractical (time, money, resources)
- Examine random samples from population
- If sample data are not consistent with hypothesis – reject

Image accessed: http://ecruzbiology.blogspot.com.au/2015_08_01_archive.html

Q. Where have all my socks gone?

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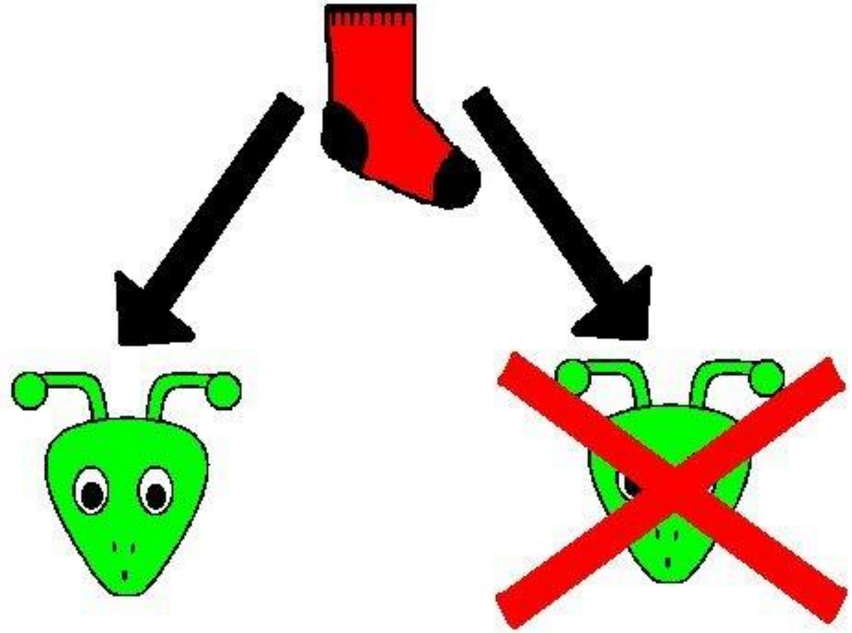


Alternate Hypothesis =

Extra-terrestrial beings have transported themselves into my house in order to steal my socks.

Null Hypothesis =

Aliens are not to blame. There is some other explanation for the disappearing socks.

Statistical Hypothesis

- Two types of statistical hypotheses

- Null hypothesis – H_0

- Alternative hypothesis – H_1 or H_a

Image accessed: http://www.clipartpanda.com/categories/delinquency-clipart

Null Hypothesis

- Represents the status quo
- The hypothesis that states there is no statistical significance between two variables in the hypothesis
- Believed to be true unless there is overwhelming evidence to the contrary
- It is the hypothesis the researcher is trying to disprove

Null Hypothesis

Null Hypothesis

Example:

It is hypothesised that flowers watered with lemonade will grow faster than flowers watered with plain water.

Null hypothesis:

There is no statistically significant relationship between the type of water used and the growth of the flowers.

Image accessed: http://beyourselfandtherestwillfollow.com/its-never-too-late-to-blossom/

Alternative Hypothesis

- Inverse of the null hypothesis
- States that there is a statistical significance between two variables
- Holds true if the null hypothesis is rejected
- Usually what the researcher thinks is true and is testing

Alternative Hypothesis

Null hypothesis:

If one plant is fed lemonade for one month and another is fed plain water, there will be no difference in growth between the two plants

Alternative Hypothesis:

If one plant is fed lemonade for one month and another is fed plain water, the plant that is fed lemonade will grow more than the plant that is fed plain water

Image accessed: http://beyourselfandtherestwillfollow.com/its-never-too-late-to-blossom/

Stating the Null and Alternative Hypothesis

Example:

- I have an assignment due for my subject. My hypothesis is that it will take an average of 6 days for me to complete the assignment. I want to test this hypothesis – that the population mean, μ, is equal to six days. To conduct the test, I gather a sample of people who have completed the assignment in the past and calculate the average number of days it took them to complete it. Suppose the sample mean is 6.1 days. The hypothesis test will tell me whether 6.1 days is significantly different from 6.0 days.

Image accessed: http://joindrpam.com/write-assignment/

Stating the Null and Alternative Hypothesis

Example:

- I have invented a golf ball that I think will increase the distance that the ball is hit off the tee by more than 20 meters. To test this hypothesis I gather a sample of golfers and calculate the mean increase in distance hit when using the golf balls I have invented.

Image accessed: http://www.sportsclipart.com/sports_clipart/a_man_hitting_a_golf_ball_with_a_club_0521-1005-1515-5654.html

Stating the Null and Alternative Hypothesis

- If the purpose is to test that the population mean is equal to a specific value (assignment example)

$H_0 : \mu = 6.0$ days

$H_1 : \mu \neq 6.0$ days

- Improvement over current products, processes or procedures (golf example)

$H_0 : \mu \leq 20$ m

$H_1 : \mu > 20$ m

Two –Tail Hypothesis Test

- Two-tail hypothesis test is used whenever the alternative hypothesis is stated as ≠

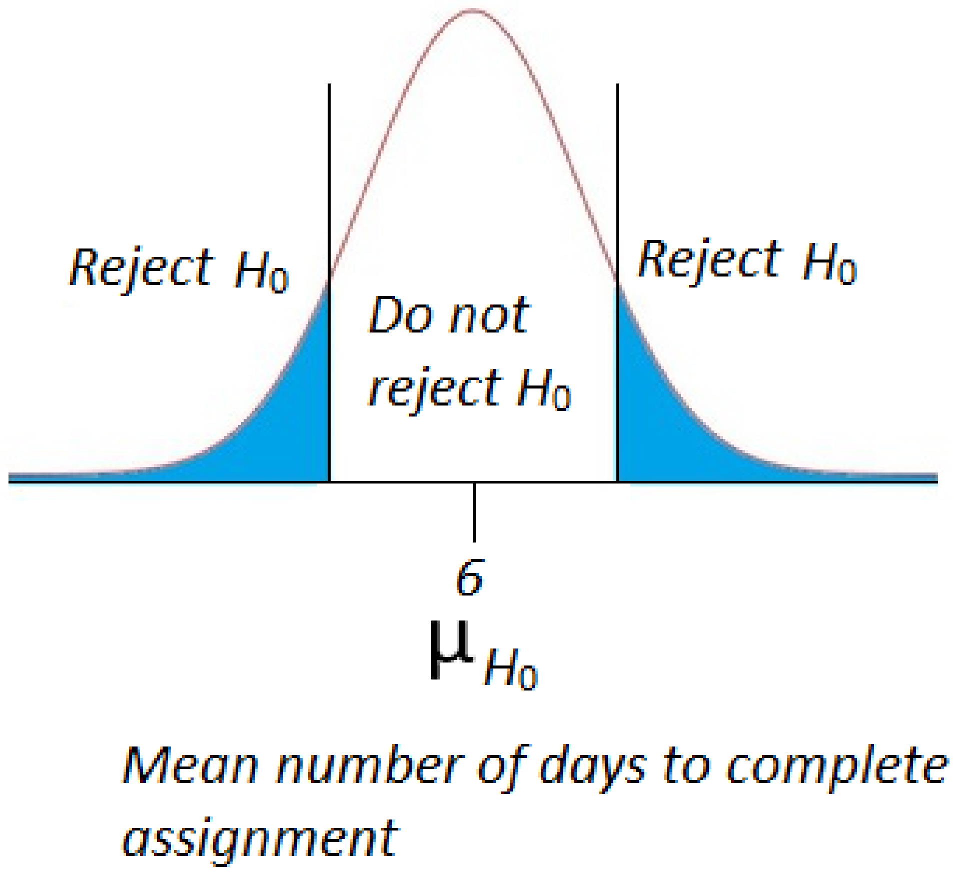
- The assignment example would require a two-tail test because the alternative hypothesis is stated as:

$H_1 : \mu \neq 6.0$ days

Image accessed: http://www.herpetofauna.co.uk/forum/two-tailed-lizard_topic4731.html

Two –Tail Hypothesis Test

- The curve represents the sampling distribution of the mean for the number of days it takes to complete the assignment

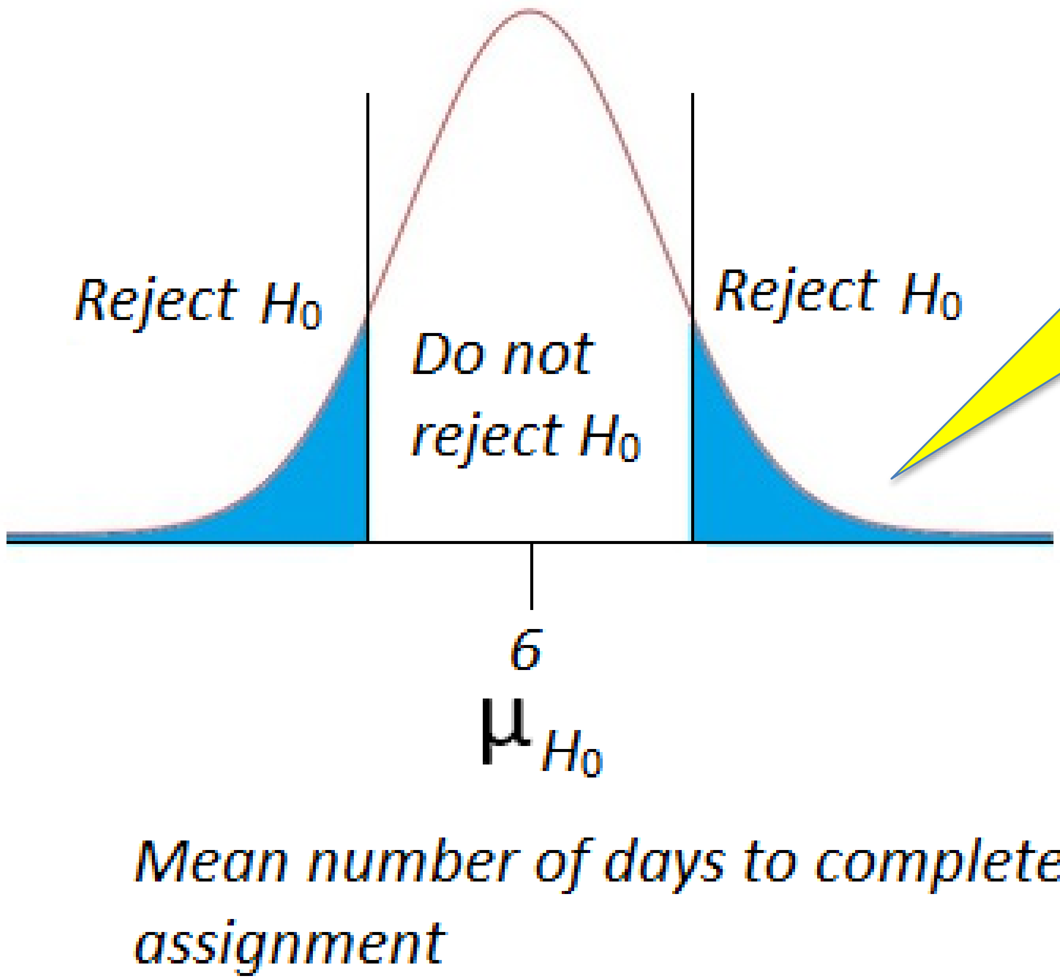


Two –Tail Hypothesis Test

Procedure:

- Collect a sample size of n, and calculate the test statistic – in this case sample mean
- Plot the sample mean on x-axis of the sampling distribution curve
- If sample mean falls within white region – we do not reject null hypothesis
- If sample mean falls in either shaded region – reject null hypothesis

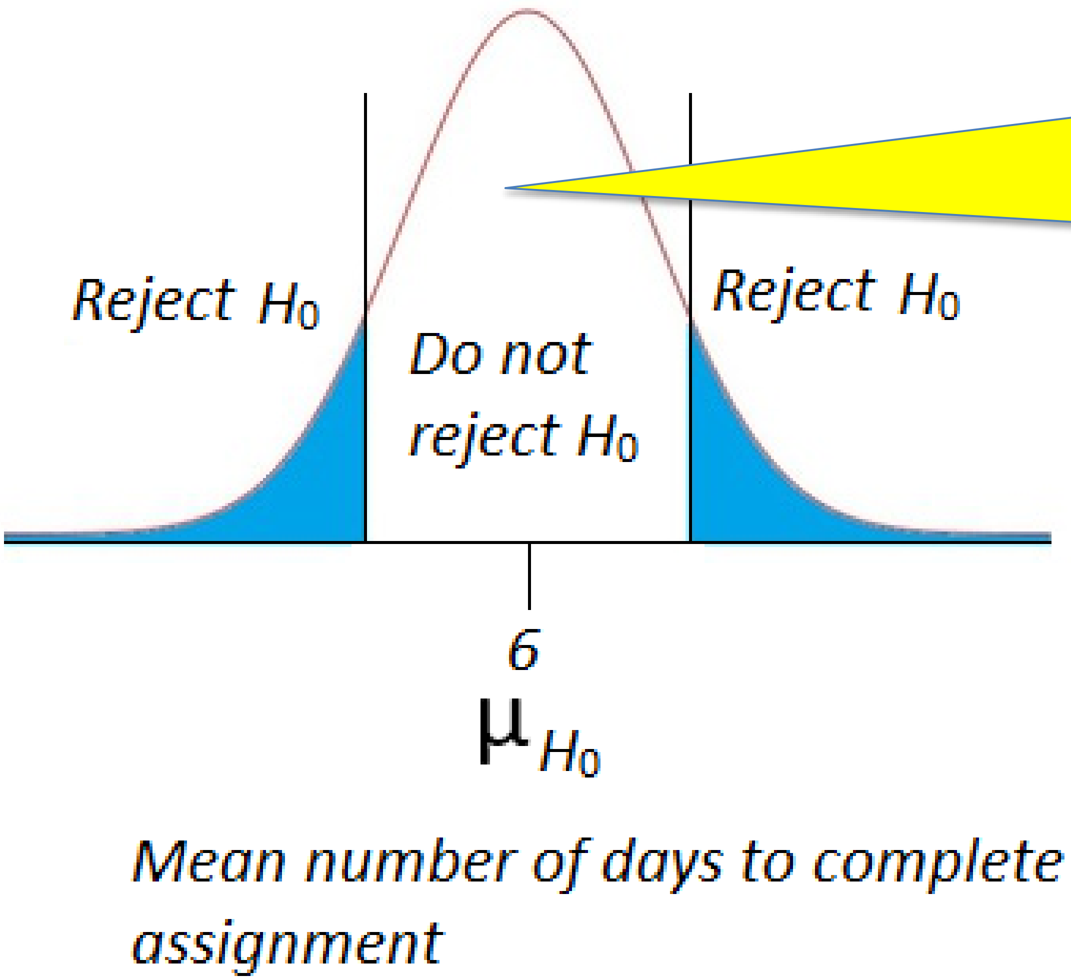
Two –Tail Hypothesis Test



We have enough evidence to support the alternative hypothesis – true population mean is not equal to 6 days

$H_1 : \mu \neq 6.0$ days

Two –Tail Hypothesis Test



We do not have enough evidence to support the alternative hypothesis – which states that the true population mean is not equal to 6 days

Two –Tail Hypothesis Test

There are only two statements we can make about the null hypothesis:

- Reject the null hypothesis
- Do not reject the null hypothesis

As conclusions are based on a sample, we do not have enough evidence to ever accept the null hypothesis.

To remember, use the analogy of the legal system. A jury or judge finds a defendant "not guilty" – they are not saying the defendant is innocent. They are saying there is not enough evidence to prove guilt.

Image accessed: http://www.clipartpanda.com/categories/courtroom-20clipart

One –Tail Hypothesis Test

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- One-tail hypothesis test is used whenever the alternative hypothesis is stated as < or >

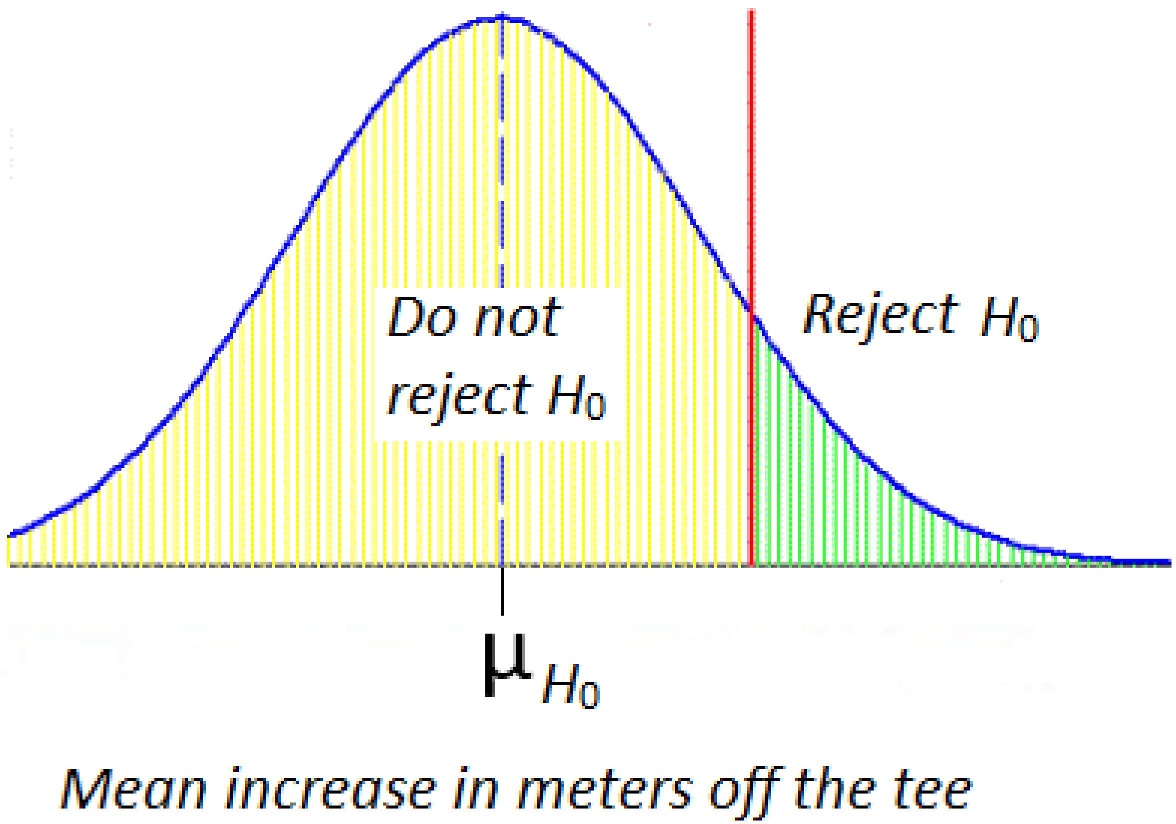
- The golf example would require a one-tail test because the alternative hypothesis is expressed as:

$$H_1 : \mu > 20 \text{ m}$$

Image accessed: http://www.suggestkeyword.com/YnVycm93aW5nIGxpemFyZHM/

One –Tail Hypothesis Test

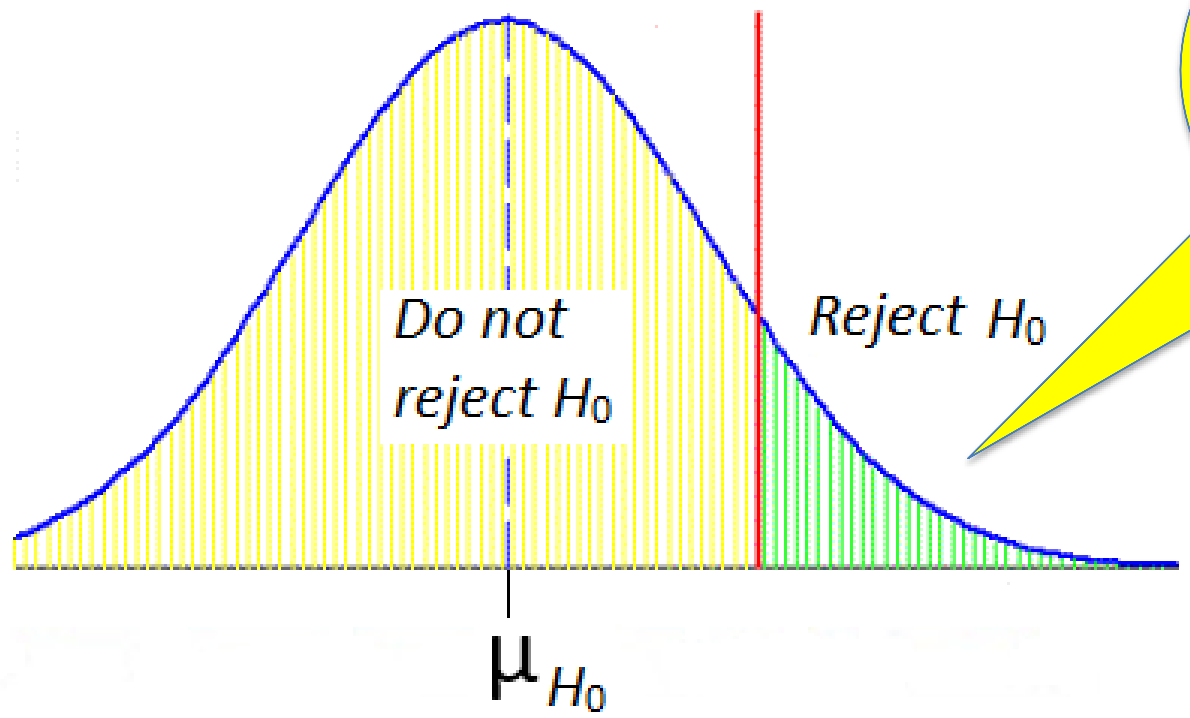
- Test and plot the sample mean, which represents the average increase in distance from the tee using the golf ball



One –Tail Hypothesis Test

- Collect a sample size of n, and calculate the test statistic – in this case sample mean
- Plot the sample mean on x-axis of the sampling distribution curve
- If sample mean falls within white region – we do not reject null hypothesis
- If sample mean falls in the shaded region – reject null hypothesis

One –Tail Hypothesis Test

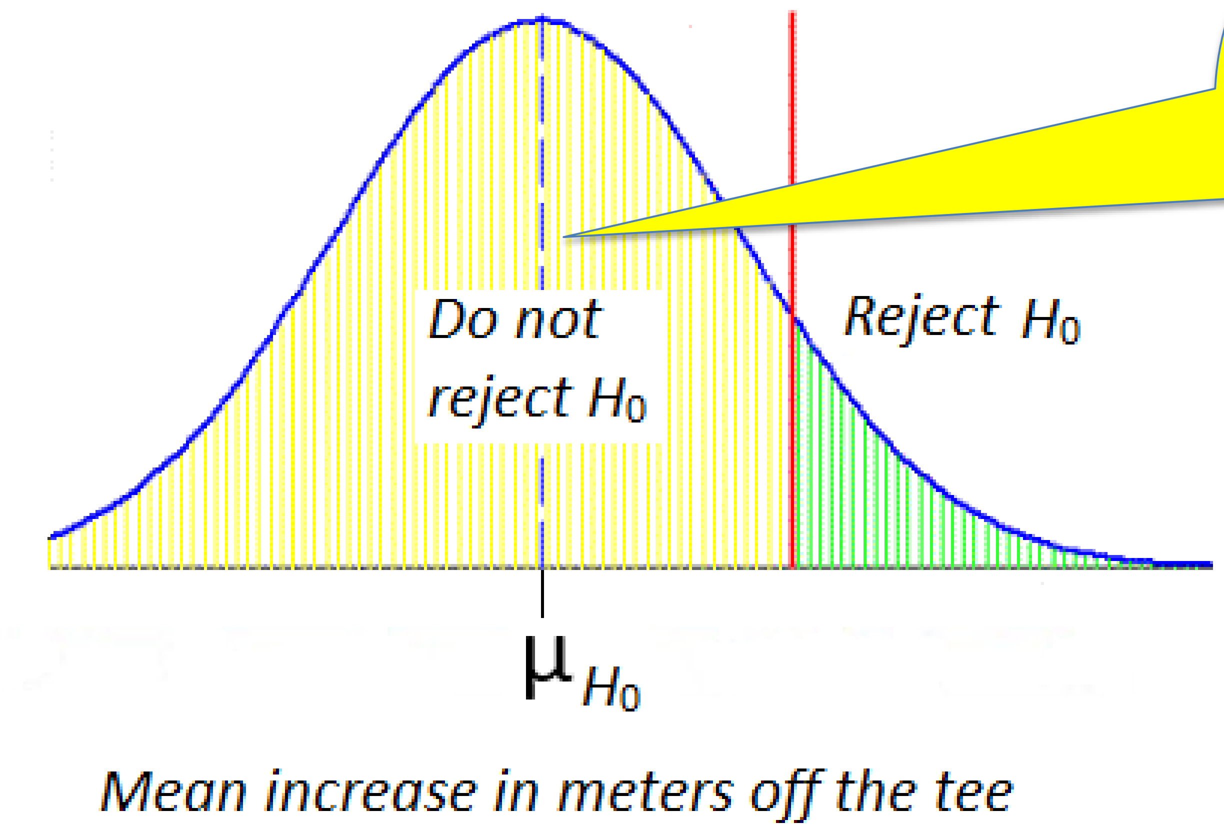


Mean increase in meters off the tee

We have enough evidence to support the alternative hypothesis – golf ball will increase distance off the tee by more than 20 m

$H_1 : \mu > 20$ m

One –Tail Hypothesis Test



We do not have enough evidence to support the alternative hypothesis – which states that the golf ball increased distance off the tee by more than 20 m

$H_0 : \mu \leq 20$ m

Type I and Type II Errors

		THE TRUTH	
		The null hypothesis (H_0) is true (H_a is false)	The null hypothesis (H_0) is not true (H_a is true)
THE DECISION THE ANALYST MAKES	Reject H_0 (support H_a)	TYPE I (α) error/ Alpha Risk/ p – value Overreacting ($1 - \alpha$) = the Confidence level of the test	Correct Decision ($1 - \beta$) Power of the test
	Fail to Reject H_0 (do not support Ha)	Correct Decision	TYPE II (β) error/ Beta Risk Underreacting

Image accessed: http://www.sixsigmadaily.com/type-i-and-type-ii-errors-in-hypothesis-testing/