Maths Refresher

Workbook 2

This module covers concepts such as:

- **Algebra concepts**
- **Basic statistics**

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Maths Refresher: Booklet 2

Contents

1. What is Algebra

Algebraic thinking spans all areas of mathematics. It involves forming and recognising number relationships, and then expressing these relationships through a symbol system. For example, in words we could say that, 'An odd number is any number that when divided by two will leave one'. We could also write this as an algebraic expression:

$$\text{"When } \frac{x}{2} \text{ has a remainder of 1, } x \text{ is an odd number."}$$

Hence, algebra provides the written form to express mathematical ideas. For instance, if I had a bag of apples that were to be shared between four people, then I could use a **pronumeral** to represent the number of apples (in this case I use the letter n), and then I can express the process of sharing n apples between four people mathematically; each person will receive $\frac{n}{4}$ apples.

Some Key Ideas:

- Zeros and ones can be eliminated. For example:
    - When we add zero it does not change the number, $x + 0 = x$ or $x - 0 = x$
      $$(6 + 0 = 6, \qquad 6 - 0 = 6)$$
    - If we multiply by one, then the number stays the same, $x \times 1 = x$ or $\frac{x}{1} = x$
      $$\left(6 \times 1 = 6, \quad \frac{6}{1} = 6\right)$$
    - Note: when we work with indices (powers) any number raised to the power zero is 1, this works because when we divide indices we subtract the index, and thus get zero, as demonstrated below:
      $$2^2 \div 2^2 = 4 \div 4 = 1 \quad therefore, \ 2^2 \div 2^2 = 2^{2-2} = 2^0 = 1$$
- Multiplicative Property: $1 \times x = x$
- Multiplying any number by one results in a product that is the number itself.
- Additive Inverse: $x + (-x) = 0$
- Any number added to its negative equals zero.
- Multiplicative Inverse: $\frac{x \times 1}{x} = 1$
- Any number multiplied by its reciprocal equals one. $x \times \frac{1}{x} = 1$; $4 \times \frac{1}{4} = 1$
- Symmetric Property: $If \ x = y \ then \ y = x$ Perfect harmony.
- Transitive Property: $If \ x = y \ and \ y = z, then \ x = z$
  For example, if apples cost \$2 and oranges cost \$2 then apples and oranges are the same price.
- It is also important to apply the **Calculation Priority Sequence** recognised by the mnemonic known as the BIDMAS/BODMAS (booklet 1, p. 6).

<u>**1. Your Turn:**</u> Use the **calculation priority sequence** to help you solve these problems:

a) $10 - 2 \times 5 + 1 =$

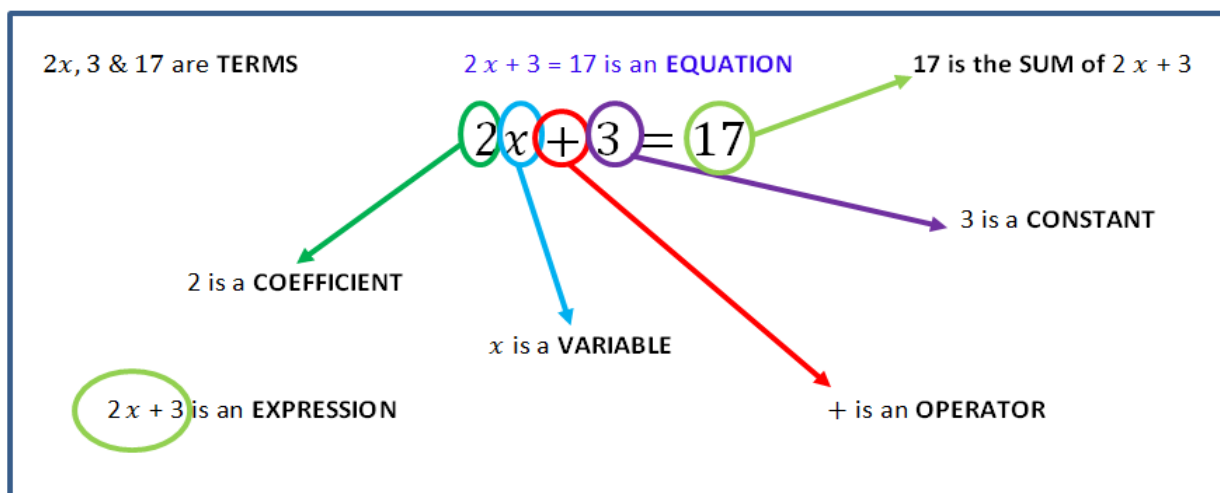c) $12 \times 2 - 2 \times 7 =$

b) $10 \times 5 \div 2 - 3 =$

d) $48 \div 6 \times 2 - 4 =$

Also apply a **golden rule**:
"What we do to one side we do to the other."

- Step One: $6a = 18$
- Step Two: $6a = 18$
  $» \div 6 \qquad \div 6$
- Step Three: $6a = (18 \div 6)$
  $» \div 6 \qquad 3$
- Step Four: $a = 3$

2. Glossary

2x, 3 & 17 are TERMS   2 x + 3 = 17 is an EQUATION   17 is the SUM of 2 x + 3

$$2x + 3 = 17$$

2 is a COEFFICIENT   3 is a CONSTANT   x is a VARIABLE   2 x + 3 is an EXPRESSION   + is an OPERATOR

Equation: Is a mathematical sentence. It contains an equal sign meaning that both sides are equivalent.

Expression: An algebraic expression involves numbers, operation signs, brackets/parenthesis and pronumerals that substitute numbers.

Operator: The operation $(+, -, \times, \div)$ which separates the terms.

Term: Parts of an expression separated by operators.

Variable: A letter which represents an unknown number. Most common is x, but it can be any symbol.

Constant: Terms that contain only numbers that always have the same value.

Coefficient: Is a number that is partnered with a variable. Between the coefficient and the variable is a multiplication. Coefficients of 1 are not shown.

Variable:	x	Operator:	$+$
Constant:	3	Terms:	$3, 2x$ (a term with 2 factors) & 17
Equation:	$2x + 3 = 17$	Left hand expression:	$2x + 3$
Coefficient:	2	Right hand expression:	17 (which is the sum of the LHE)

2. Your Turn:

If $x = 6$, then what is the value of?

a) $4x + 3$

b) $9 - \frac{x}{2}$

c) $\frac{x}{3} + 2$

d) Are any of these equations? Explain your answer.

e) In a), what would you refer to the x as? What about the 9 in b)? And the + in c)?

Watch this short Khan Academy video for further explanation:
"Expression terms, factors and coefficients"
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-expressions-and-variables/cc-6th-writing-expressions/v/expression-terms-factors-and-coefficients
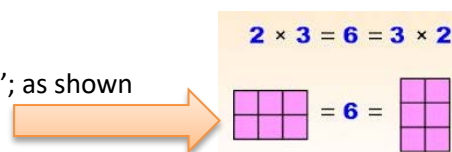
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3. Addition and Multiplication Properties

The Commutative Property of Addition: $x + y = y + x$

- ° The order that numbers are added does not affect the 'sum'; for example, $2 + 3 = 3 + 2$

The Commutative Property of Multiplication: $x \times y = y \times x$

- ° The order that numbers are multiplied does not affect the 'product'; as shown

$2 \times 3 = 6 = 3 \times 2$

$= 6 =$

The Associative Property of Addition: $(x + y) + z = x + (y + z)$

- ° When adding two or more numbers, the order in which you add numbers does not matter. The significance of this property is that it is possible to **regroup** the numbers so that the order of calculation can be established. For example, while $(3 + 4) + 6 = 13$ and $3 + (4 + 6) = 13$, the brackets signify which group of digits should be added together first.
- ° Understanding this concept can make adding numbers easier, because we can group 'nice' numbers together, for example, pair numbers that equal 10, $(4 + 6)$.

The Associative Property of Multiplication: $(x \times y) \times z = x \times (y \times z)$

- ° When multiplying two or more numbers, the order or multiplication does not matter. The significance of this property is that it is possible to **regroup** the numbers so that the order of calculation can be established. Take a minute to compare the associative property of multiplication to the commutative property of multiplication. How do these properties differ? You might have noticed that in commutative property the numbers are moved around – not regrouped.

The Distributive Law: Multiplication distributes over addition or subtraction through the brackets (parentheses).
For example: $x(y + z) = xy + xz$

- ° Allows for a separate term to be multiplied by other terms inside the brackets individually and still get the same result. Thus every term inside the brackets is multiplied by the term that is outside of the brackets – think of this as expanding a mathematical expression.
- ° For example:
$$2(3 + 4) = 2 \times 3 + 2 \times 4$$
$$2(7) = 6 + 8$$
$$14 = 14$$
- ° Remember the calculation priority sequence (BIMDAS) too, because if we simply worked from left to right, the answer would be 32 and thus incorrect.

Distributive Law can work for division too – **but** only from right to left.

- ° For example:
$$(12 + 8) \div 2 = 10$$
$$(12 \div 2) + (8 \div 2) = 6 + 4 = 10$$

- ° However, this will not work for $40 \div (3 + 2)$
$$40 \div (3 + 2) = 40 \div 5 = 8 \quad ✓$$
$$whereas (40 \div 3) + (40 \div 2) = 13\frac{1}{3} + 20 = 33\frac{1}{3} \quad ✗$$

3. Your Turn:

a) Rewrite $3 \times 2 \times x$ by using the 'commutative property' of multiplication.
b) Rearrange $2(4x)$ in using the 'associative property' of multiplication.
c) Rewrite $8(2 + x)$ using the 'distributive law'.

4. Collecting Like Terms

Algebraic thinking involves simplifying problems to make them easier to solve. Often the sight of variables can raise mathematical anxieties, yet once we understand a little more, the anxieties can dissipate.

Mathematical anxiety is common and the best way to relieve it is to recognise what we do know; then begin to work methodically to solve the problem. For instance, in the equation below, we can look at the equation as bits of information so as the equation becomes easier to solve. In short, we simplify the problem into a smaller problem and this is done by **collecting like terms**.

$$7x + 2x + 3x - 6x + 2 = 14$$

A 'like term' is a term which has the same variable [which may also have the same exponent/indices (see booklet 1, p. 20)], only the coefficient is different. For example, in the equation above we can see that we have four different coefficients $(7, 2, 3, \& 6)$ with the same variable, x, yet there are no exponents to consider. Once identified, we can collect these like terms: $7x + 2x + 3x - 6x$

Take a moment to read this **expression** (note that above the paragraph the expression was actually an **equation**, why?), what is it telling you? Can you explain this expression to the person sitting next to you?

We can now view the coefficients as separate from the variable: we can add and subtract the coefficients to get six:

$$7 + 2 + 3 - 6 = 6$$
$$\text{Thus, } 7x + 2x + 3x - 6x = 6x$$

And, the original equation simplifies to $6x + 2 = 14$ (adding in the constant)

Once simplified, it is easier for us to solve the equation:

$$6x + 2 - \mathbf{2} = 14 - 2$$
$$6x = 12$$
$$6\,x(\div 6) = 12 \div 6$$
$$x = 2$$

Example Problem: Collect the like terms and simplify:

$$5x + 3xy + 2y - 2yx + 3y^2$$

Step 1: Recognise the like terms: (note: xy is the same as yx; commutative property)

Step 2: Arrange the expression so that the like terms are together (remember to take the operator with the term).

$$5x + 2y + 3xy - 2yx + 3y^2$$

Step 3: Complete the operation:

$$5x + 2y + 1xy + 3y^2$$
$$\text{Note: a coefficient of 1 is not usually shown}$$
$$\therefore 5x + 2y + xy + 3y^2$$

4. Your Turn:

Collect the like terms using the steps above:

a) $3x + 2y - x$

b) $2x^2 - 3x^3 - x^2 + 2x$

c) $3m + 2n + 3n - m - 7$

d) $4(x + 7) + 3(2x - 2)$

e) $3(m + 2n) + 4(2m + n)$

f) $\frac{x}{3} + \frac{x}{4}$

5. Simplifying Expressions: Using 'Expansion'

To simplify expressions involves 'expanding' or 'factorising'. This section helps you to investigate the concept of what it means to expand an expression. When we expand an expression, we are removing the brackets – brackets are often referred to as the grouping symbols. '**Expansion'** involves applying the distributive law.

Let's look at an example: $x(6 + 9)$ As we know from the distributive law, the $'x'$ outside of the brackets is multiplied **through** the brackets. So we can express $x(6 + 9)$ as: $6x + 9x$. In this expression we have two like terms and thus we can simplify further to $15x$.

So if we multiply two numbers together, the order in which we multiply is irrelevant; **commutative property**
For example: Simplify $4(3x)$

This could be expressed as $4 \times (3 \times x)$ and also as $(4 \times 3) \times x$
Therefore, we can simplify to $12x$

5A. Your Turn:

Simplify these expressions using expansion:

a) $x(4 + 3)$
b) $x(3 - 1)$
c) $x(8 + 6)$

Sometimes there may be **nested** grouping symbols. This happens when there are two sets of brackets – one is *nested* inside the other. This means that the operations in the inner set must be worked first. Three example problems are provided below, none of which include pronumerals, so that the process of solving a series of nested brackets is more visible.

Example Problems:

1) $20 - [3(14 - 10)] =$
$20 - [3 \times (14 - 10)] =$
$20 - [3 \times 4] =$
$20 - 12 = 8$

2) $4[(6 + 3) \times 5] =$
$4[9 \times 5] =$
$4(45) = 180$

3) $\frac{(3+21)}{[12-(11-7)]} = \frac{(3+21)}{(12-4)}) =$
$\frac{24}{8} = 3$

5B. Your Turn:

Use what you have learnt about expansion and nested brackets to solve these equations:

d) $8[9 - (5 + 2)] =$

e) $2[4 + 5(6 - 5)] =$

f) $2[3(13 - 8) \times 4] =$

g) $\frac{2(4+6)}{2+3} =$

h) $\frac{(12 \times 4)}{4(4+2)} =$

i) What is the missing number of this equation? $5 + \{4[\_ + 3(7 + 2)]\} = 125$

Watch this short Khan Academy video for further explanation:
"Combining like terms and the distributive property"
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-variables-expressions/cc-7th-manipulating-expressions/v/combining-like-terms-and-the-distributive-property

6. Simplifying Expressions with Exponents

Now let's work with an algebraic expression with brackets and exponents. We will simplify using expansion, eventually incorporating terms with exponents:

- ° To begin, simplify $(3x)(6x)$
- ° This is essentially: $(3 \times x) \times (6 \times x)$
- ° We can change the order $(3 \times 6) \times (x \times x)$
- ° Therefore, further simplify to $18x^2$

Work strategically:

- ° Is $2x^2$ the same as $(2x)^2$?
- ° Let's investigate:

$\mathbf{2x^2}$ is $2 \times x \times x$ $(\mathbf{2x})^2$ is $(2 \times x) \times (2 \times x)$

so if x was 5; $2 \times 5 \times 5 = 50$ so if x was 5; $10 \times 10 = 100$

Therefore, $2x^2$ is not the same as $(2x)^2$

This example demonstrates how the commutative property can be used:

$$6x \times 2y \times 3xy = 6 \times x \times 2 \times y \times 3 \times x \times y$$
$$= 6 \times 2 \times 3 \times (x \times x) \times (y \times y)$$
$$= 36x^2 y^2$$

…and now we also apply the 'INDEX LAWS' to simplify an expression including terms with exponents (indices). Simplify $5x^2 \times 6x^5$

- ° So we can say that we have $(5 \times 6) \times (x \times x) \times (x \times x \times x \times x \times x)$
- ° This is $30 \times x^{(2+5)}$
- ° Therefore, $30x^7$

There are steps to follow which may assist your reasoning process:

- Simplify expressions that have grouping symbols first working from the innermost to the outer. As you do this, apply the Calculation Priority Sequence (BIDMAS/BOMDAS).
  - o Simplify powers
  - o Multiply in order from left to right
  - o Add and subtract in order from left to right.
- Then work backwards to check

Example problems:

What is the difference between the expressions: $(6x)(5x)$ and $(6x) + (5x)$?

$$(6x)(5x) = 30x^2 \text{ whereas } (6x) + (5x) = 11x$$

One more equation: $(-5a^2)(-2a) = 10a^3$

Here we apply the index law principle that $a^2 \times a^1 = a^3$ as well as the rule that a negative multiplied by a negative is positive: $(-5) \times (-2) = 10$

6. Your Turn: Simplify the following…

a) $2x + 7x + 11x =$ c) $6x^2 - 5x^2 =$

b) $4xy + 7xy =$ d) $5x^2 + 7x + 3x =$

7. Solving Equations

An **equation** states that two quantities are equal, and may contain an **unknown quantity** that we wish to find; in the equation, $5x + 10 = 20$, the unknown quantity is x.

- To solve an equation means to find all values of the unknown quantity so that they can be substituted to make the left side **equal** the right side.

- Each such value is called a **solution**, or alternatively a **root** of the equation. In the example above, the solution is $x = 2$ because when 2 is substituted, both the left side and the right side equal 20 ∴ the sides balance. The value $x = 2$ is said to **satisfy** the equation

- Sometimes we might need to rearrange or **transpose** the equation to solve it; essentially, what we do to one side we do to the other.

(Croft & Davison, 2010, p. 109)

Let us look at an example: $2x + 6 = 14$

> In words: What number can we double then add six so we have a total of fourteen?

> We can apply the following two principles to solve the equation:

> 1. Work towards the variable.
> 2. "What we do to one side we must do to the other"

> So let's solve the problem …

> Step 1: The constant, 6, is our first target. If we take 6 from both sides, we create the following equation.

$$2x + 6 - 6 = 14 - 6 \quad \text{(The opposite of } +6 \; is \; -6)$$
$$2x = 8 \quad\quad (+6 - 6 = 0)$$

> Step 2: The only number left on the same side as the variable is the coefficient, 2. It is our second target. If we divide both sides by two, we create the following equation: (Note: between the 2 and the x is an invisible multiplication sign)

$$\frac{2x}{2} = \frac{8}{2} \quad \text{(The opposite of } \times 2 \; is \div 2)$$

$$x = 4$$

> Step 3: Check. If we substitute a 4 into the equation we have:

$$2 \times 4 + 6 = 14$$
$$8 + 6 = 14$$
$$14 = 14 \quad \text{(We are correct)}$$

NOTE: **To remove a constant or coefficient, we do the opposite on both sides.**

Opposite of $\times$ is $\div$
Opposite of $+$ is $-$
Opposite of x^2 is $\sqrt{x}$

Example Problems:

1. Solve for J

$$3J - 5 = 16 \text{ (Target 5 then 3)}$$

$$3J - 5 + 5 = 16 + 5 \text{ (Opposite of -5 is +5)}$$

$$3J = 21$$

$$\frac{3J}{3} = \frac{21}{3} \text{ (Opposite of x3 is /3)}$$

$$J = 7$$

(Check: $3 \times 7 - 5 = 16$ ✓)

2. Solve for T

$$\frac{3T}{12} - 7 = 6 \text{ (Target 7 then 12 then 3)}$$

$$\frac{3T}{12} - 7 + 7 = 6 + 7$$

$$\frac{3T}{12} = 13$$

$$\frac{3T}{12} \times 12 = 13 \times 12$$

$$3T = 156$$

$$\frac{3T}{3} = \frac{156}{3}$$

$$T = 52$$

(Check: $3 \times 52 \div 12 - 7 = 6$ ✓)

7. Your Turn:

Transpose to make x the subject: you may like to leave f), g) and h) until you have completed the next section.

a) $y = 3x$

b) $y = \frac{1}{x}$

c) $y = 7x - 5$

d) $y = \frac{1}{2}x - 7$

e) $y = \frac{1}{2x}$

f) $y = 13(x - 2)$

g) $y = x(1 + \frac{1}{x})$

h) $y = a + t(x - 3)$

Solve the following:

i) $5x + 9 = 44$

j) $\frac{x}{9} + 12 = 30$

k) $3y + 13 = 49$

l) $4x - 10 = 42$

m) $\frac{x}{11} + 16 = 30$

8. Multiples and Factors

This section will provide you with a refresh on factors and multiples. These concepts relate to understanding fractions, ratios and percentages, and they are integral to factoring when solving more difficult problems.

Multiples, common multiples and the LCM

- A **multiple** of a given a number, into which that number can be divided exactly.

    For example: multiples of 5 are 5, 10, 15, 20, 25, 30…

- A **common multiple** is a multiple in which two or more numbers have in common.

    For example: 3 and 5 have multiples in common 15, 30, 45…

- The **lowest common multiple** LCM is the lowest multiple that two numbers have in common.

    For example: 15 is the LCM of 3 and 5

Factors, common factors and the HCF

- **Factor** – a whole number that can be multiplied a certain number of times to reach a given number.

    3 is factor of 15 because it can be multiplied by 5 to get 15

- **A common factor** is a factor that two or more numbers have in common.

    3 is a common factor of 12 and 15

- The **highest common factor – HCF**

    What is the HCF of 20 and 18?

    The factors of 20 (1, 2, 4, 5, 10, 20) and 18 (1, 2, 3, 6, 9, 18)

    The common factors are 1 and 2 and

    The HCF is 2

- **Proper factors:** All the factors apart from the number itself

    18 (1, 2, 3, 6, 9, 18) 1, 2, 3, 6, 9 are proper factors of 18

- **Prime number**: Any whole number greater than zero that has exactly two factors – itself and one

    2, 3, 5, 7, 11, 13, 17, 19…

- **Composite number:** Any whole number that has more than two factors

    4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20…

- **Prime factor**

    A factor that is also a prime number

- **Factor tree**

    A tree that shows the prime factors of a number

As the factor trees show, some numbers have many factors. This is a helpful way to find the factors of a given number. A more effective method for clarity when thinking and reasoning mathematically is to list all of the factors.

8. Your Turn:

List all of the factors of each of the following numbers:

a) 14_____

b) 134_____

c) 56_____

d) 27_____

e) 122_____

Watch this short Khan Academy video for further explanation:

"Finding factors and multiples"

https://www.khanacademy.org/math/pre-algebra/factors-multiples/divisibility_and_factors/v/finding-factors-and-multiples

9. Factorising and Expanding

The distributive law is commonly used in algebra. It is often referred to as either 'expanding the brackets' or 'removing the brackets'.

- **Expanding** involves taking what is outside of the brackets and moving it **through** the brackets. For example:
  $3(x + 2)$ can be expanded to $3 \times x + 3 \times 2$ which is $3x + 6$
  This means that: $3(x + 2) = 3x + 6$

- Working in the opposite direction is called **factorisation** and this process is slightly more difficult.
- Factorisation requires finding the highest common factor. (The HCF then sits outside of the brackets.)
- Let's look at $3x + 6$, we can factorise this back again from the expanded form.
  - First we need to find a common 'factor'; we can see that x can be divided by 3 and 6 can be divided by 3. Our common factor is 3. Now we put this 3 outside of the brackets (because we have divided each of the terms by three) to be multiplied by everything inside of the brackets.
  - So we have $3(x + 2)$

- Another factorisation example:
  - $5x + 15x^2 - 30x^3$ has three terms $5x$ and $15x^2$ and $30x^3$
  - All three terms contain x and can be divided by 5 $\therefore$ $5x$ is a common factor.
  - Next we divide each term by $5x$.
  - $5x \div 5x = 1$; $15x^2 \div 5x = 3x$; and $30x^3 \div 5x = 6x^2$
  - We end up with: $5x(1 + 3x - 6x^2)$.
- Now we can do the reverse and practise expanding the brackets.
  - So expand $5x(1 + 3x - 6x^2)$
  - Multiply each term inside the brackets by $5x$
  - $5x \times 1 = 5x$; $5x \times 3x = 5 \times 3 \times x \times x = 15x^2$; and $5x \times 6x^2 = 5 \times 6 \times x \times x \times x = 30x^3$
  - Now we have expanded the brackets to get $5x + 15x^2 - 30x^3$
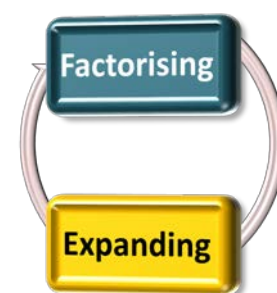
Example Problems:

Remove the Brackets:

  - $5x(2 + y)$ expands to $(5x \times 2) + (5x \times y)$ which is simplified to $10x + 5xy$
  - $-x(2x + 6)$ expands to $(-x \times 2x) + (-x \times 6)$ which is simplified to $-2x^2 - 6x$

Factorise:

  - $3x + 9x - x^2$ has only the variable x in common $\therefore$ $x(3 + 9 - x)$
  - $12x^3 + 4x^2 - 20x^4$ has $4x^2$ as the highest common factor $\therefore$ $4x^2(3x + 1 - 5x^2)$

9. Your Turn:

Expand the first two and then factorise back again, then factorise the next two and then expand for practise:

a) Expand $2y^2(3x + 7y + 2)$

b) Expand $-1(3x + 2)$

c) Factorise $3x + 6x^2 - 9x$

d) Factorise $24x + 42xy - 60x^3$

10. Function Notation

Most of the equations we have worked with so far have included a single variable. For example, $2x + 5 = 11$, x is the single variable. Single variable equations can be solved easily. Yet, in real life you will often see questions that have two variables:

The cost of fuel is equal to the amount of fuel purchased and the price per litre $1.50/L.

The mathematical equation for this situation would be:

$$Cost = Amount \times PPL \text{ (price per litre)}$$
$$\therefore C = A \times 1.5$$

If the amount of fuel is 20 litres, the cost can be calculated by using this formula:

$$C = 20 \times 1.5$$
$$\therefore C = 30$$

Another way to describe the above scenario is to use **function notation**. The cost is a **function** of amount of fuel.

If we let x represent the amount, then the cost of fuel can be represented as $f(x)$

Therefore: $f(x) = 1.5x$. We can now replace x with any value.

If $f(x) = 1.5x$ then solve for $f(3)$

This means replace x with the number 3.

$$f(3) = 1.5 \times 3$$
$$\therefore f(3) = 4.5$$

Example Problems:

1. $f(x) = 5x + 7$ Solve for $f(4)$ and $f(-2)$

$$f(4) = 5 \times 4 + 7 \qquad\qquad f(-2) = 5 \times -2 + 7$$
$$\therefore f(4) = 27 \qquad\qquad\qquad \therefore f(-2) = -3$$

2. $f(x) = \frac{3}{x} + x^2$ Solve for $f(6)$ and $f(-9)$

$$f(6) = \frac{3}{6} + 6^2 \qquad\qquad f(-9) = \frac{3}{-9} + (-9)^2$$
$$\therefore f(6) = 36.5 \qquad\qquad \therefore f(-9) = 80\frac{2}{3}$$

10. Your Turn:

a) $f(x) = 6x + 9$ Solve for $f(4)$ and $f(0)$

b) $f(x) = x^2 + \frac{6}{x}$ Solve for $f(6)$ and $f(-6)$

Watch this short Khan Academy video for further explanation:
"Evaluating with function notation"

KHAN ACADEMY

11. The Number Line

A basic tool in maths is the number line, a visual representation of all the positive and negative numbers. It has an Origin of Zero and an arrow at each end to show that the numbers continue indefinitely. This was explored briefly in the section on numbers when dealing with negative and positive integers. It is important to remember that positive numbers are to the right of zero and negative numbers are to the left of zero.
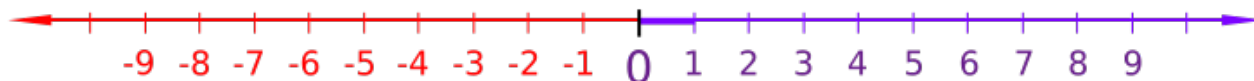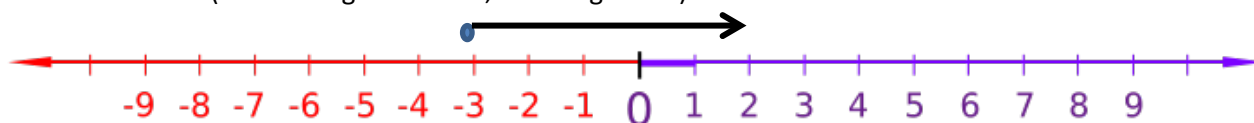
- Using the number line is based on some rules:
  - **Addition is movement to the right**
  - **Subtraction is movement to the left**
  - **Adding a negative number requires moving to the left**
  - **Subtracting a negative number requires moving to the right**

Example Problems:
1. $5 - 7 = -2$ (start at five, move left seven.
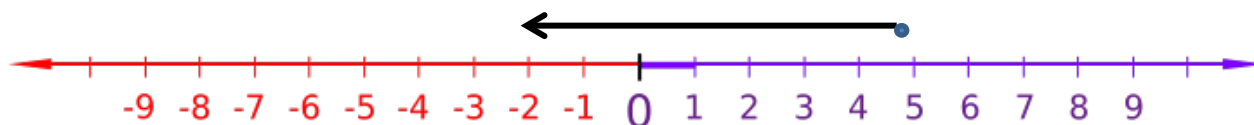
2. $-3 + 5 = 2$ (start at negative three, move right five)

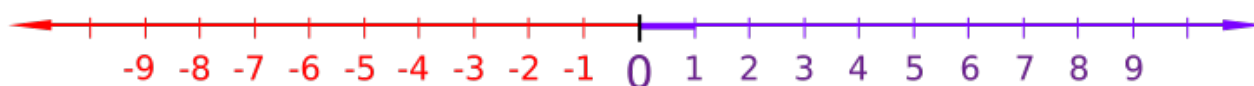3. $-2 + (-4) = -6$ (start at negative two, move four left)
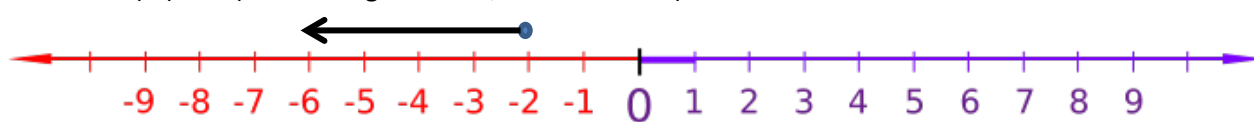
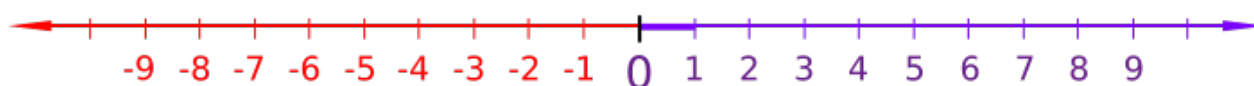4. $-5 - (-8) = 3$ (start at negative five, move eight right)
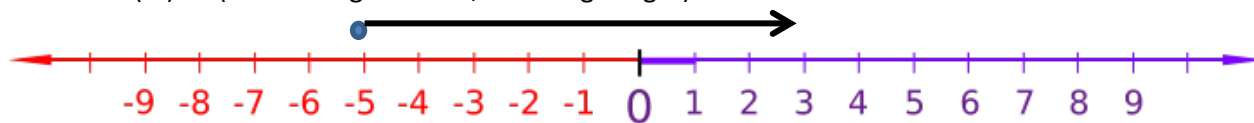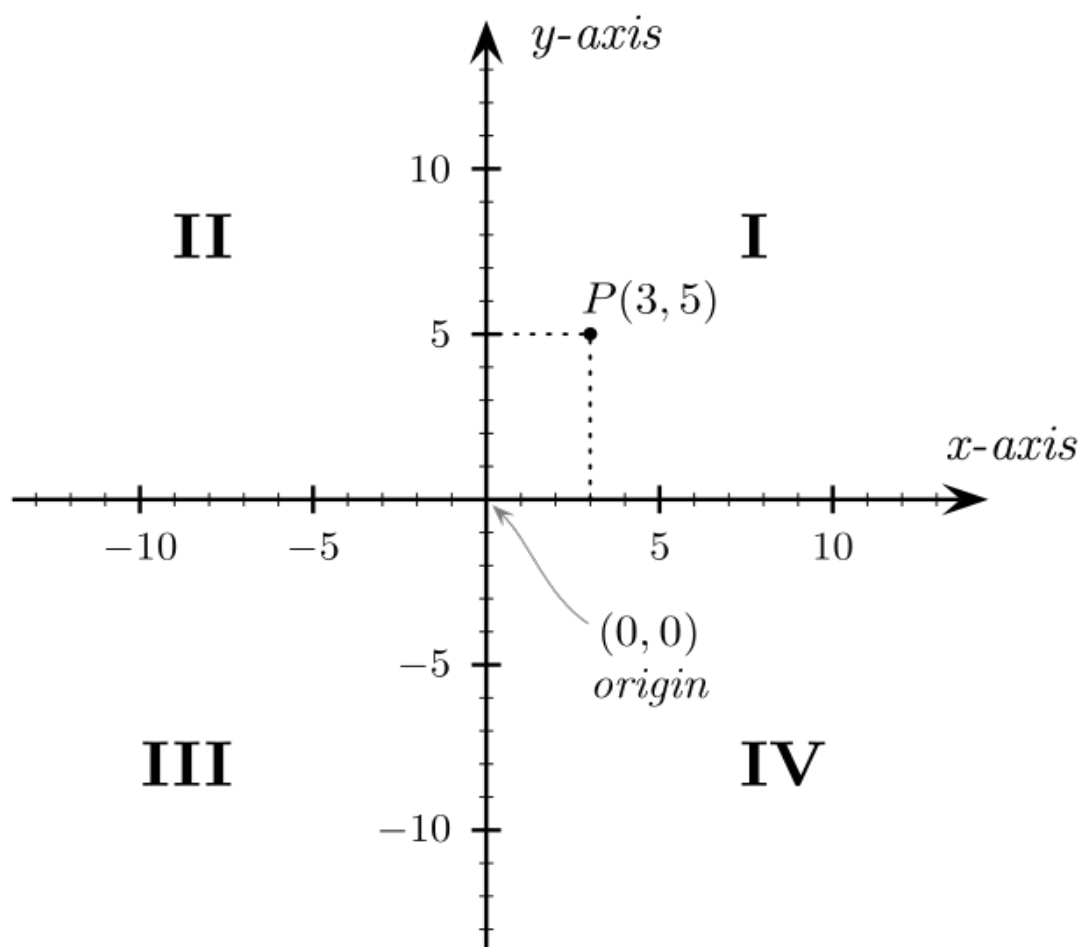
11. Your Turn:
1. $-6 + 9 =$

2. $2 - (-5) =$

12. Cartesian Plane

The Cartesian plane is an extension of the number line in two directions. It is has a horizontal number line called the x axis and a vertical number line called the y axis.

- The two axes divide the page into four quadrants numbered in an anticlockwise direction.
    - **Quadrant one** is at the top right corner and contains all positive values. This is the quadrant used for the majority of graphs.
    - **Quadrant two** is the top left corner and contains negative x values and positive y values
    - **Quadrant three** is the bottom left and contains all negative values.
    - **Quadrant four** is the bottom right quadrant and contains positive x values and negative y values.
- The point where the two number lines pass is the origin and has a value of zero for both x and y values.

12. Your Turn:

Given that the coordinates for P quadrant one are (3,5), write the coordinates for P if it was the same distance from the origin in the quadrants:

      a) II   b) III   c) IV

Watch this short Khan Academy video for further explanation:
**"Coordinate plane: quadrants"**

13. Coordinates

- The power of the Cartesian plane is the ability to locate points and to display these easily.
- Coordinates are a pair of numbers which give the location of a point on the Cartesian plane.
- The most important rule with coordinates is that the x axis value is written first.

Coordinate	x value	y value	Quadrant
$A(2, 3)$	2	3	I
$B(-2, 4)$	-2	4	II
$C(-1, -3)$	-1	-3	III
$D(4, -4)$	4	-4	IV

13. Your Turn:

a) Plot the points A, B, C, and D from the table above on this Cartesian plane.
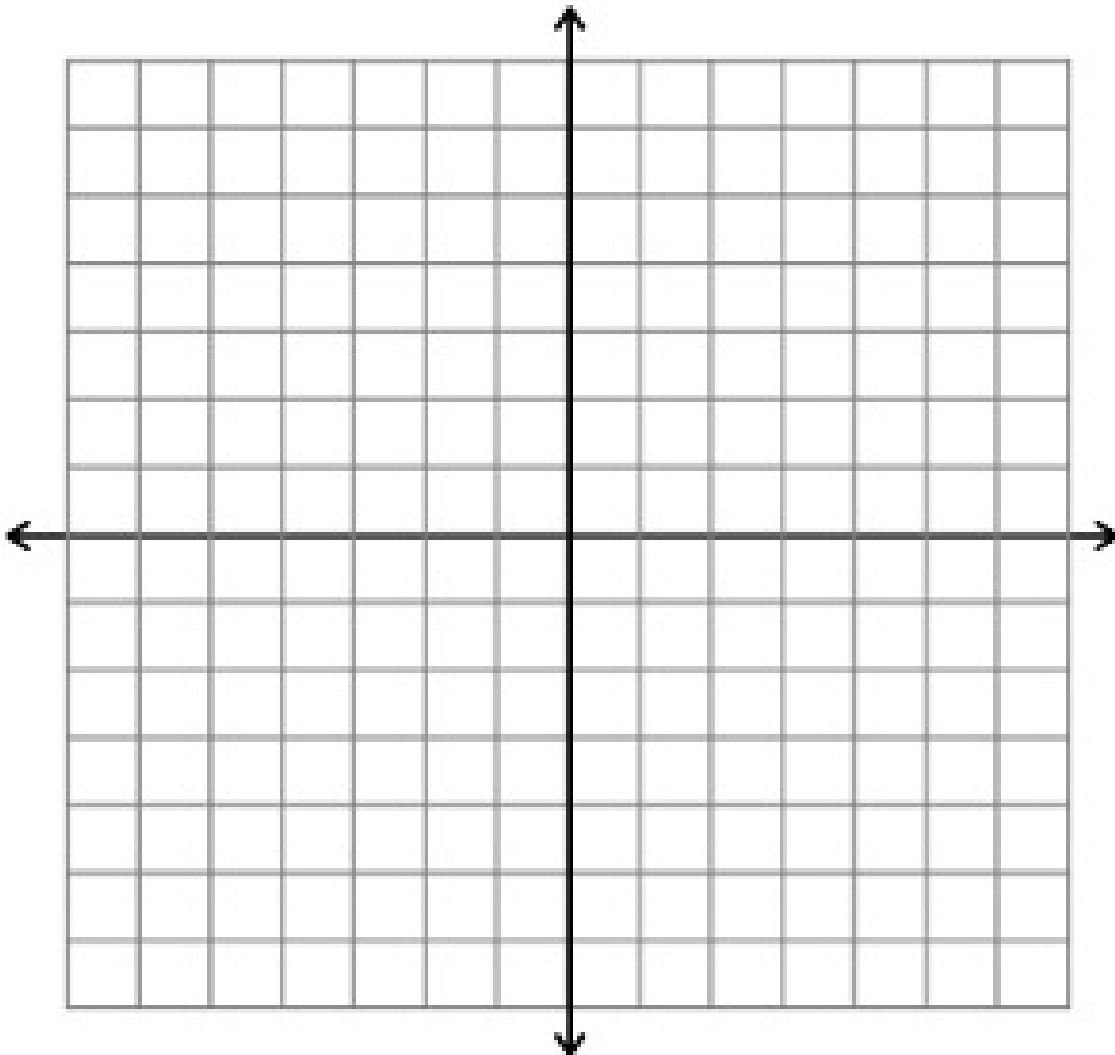
14. Linear Patterns

Cartesian Planes are often used to plot data points to see if there is a pattern in the data. A common method is using a table like the one below; the coordinates are plotted on the graph.

x value	-3	-2	-1	0	1	2	3
y value	6	4	2	0	-2	-4	-6
Coordinate	(-3, 6)	(-2, 4)	(-1, 2)	(0, 0)	(1, -2)	(2, -4)	(3, -6)
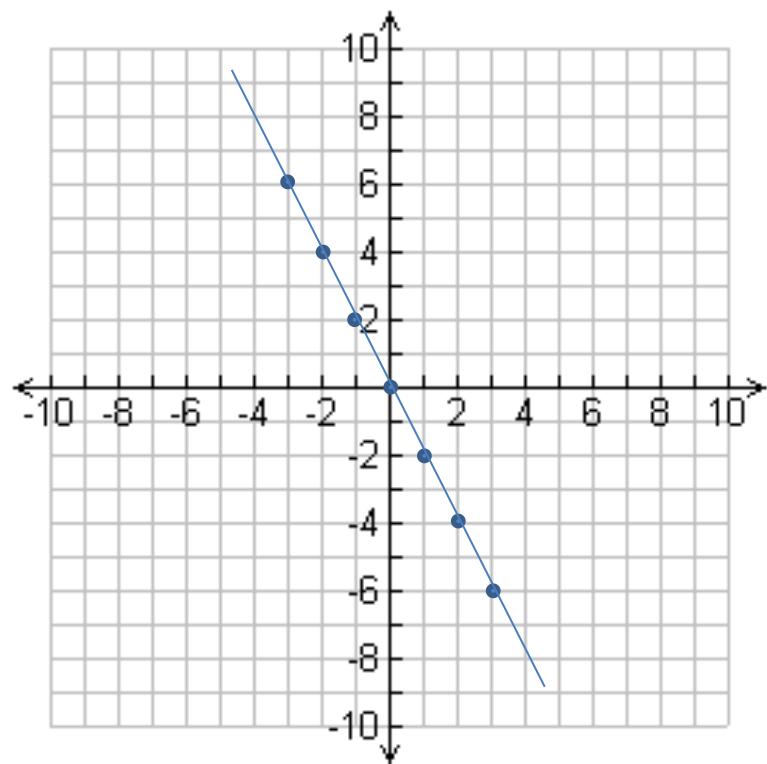
- From the graph, we can describe the relationship between the x values and the y values.
  - Because a straight line can be drawn through each point, we can say that there is a **linear** relationship in the data.
  - The graph goes 'downhill' from left to right, which implies the relationship is negative.

  So we can deduce that the relationship is **a negative linear relationship.**

Summary:

|		
Positive Linear Relation	Negative Linear Relationship	Non Linear Relationship

15. Gradient

Looking at the two lines plotted on the graph below, we can see that one line is steeper than the other; this steepness is called gradient and has the symbol m.

The formula to calculate gradient is:

$$gradient = \frac{difference\ in\ y\ coordinates}{difference\ in\ x\ coordinates}$$

$$\therefore \quad m = \frac{\Delta y}{\Delta x}$$

Often simplified to: $m = \frac{Rise}{Run}$
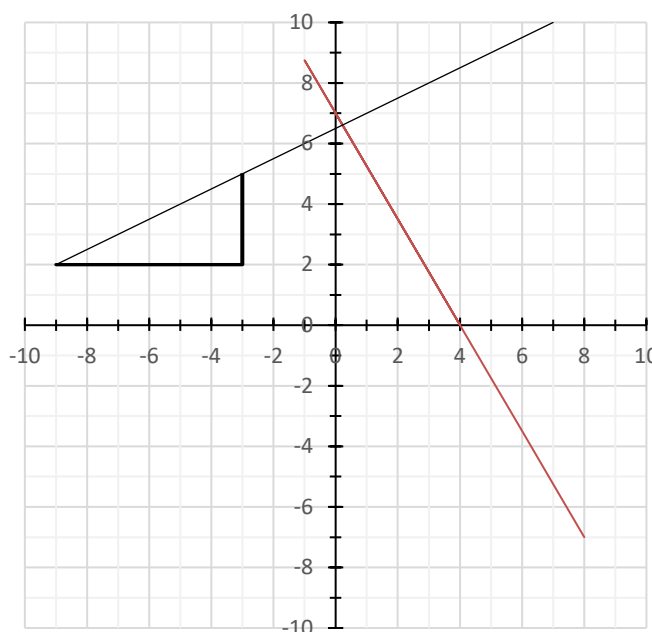
To use the formula we draw a right angled triangle on the line (trying to use easy values to work with).

On this triangle we have marked two y coordinates at 5 and 2; and two x coordinates at $-3, -9$.

Thus, looking at the coordinates marked by the triangle, we can say that there is a:
- **rise** of 3: $difference\ in\ y\ coordinates\ (5 - 2 = 3)$ and
- **run** of 6: $difference\ in\ x\ coordinates\ -3 - (-9) = 6$

so $m = \frac{Rise}{Run} = \frac{3}{6} = \frac{1}{2} = 0.5$

$\therefore$ The solid line has a positive gradient of 0.5. (It is positive because it is 'uphill'.)

14. Your Turn:

a) Calculate the gradient of the red line.

$$gradient = \frac{difference\ in\ y\ coordinates}{difference\ in\ x\ coordinates}$$

16. Intercept

- Look at the three lines below, they each have the same slope, yet they are obviously different graphs. To describe how the lines differ from each other, we describe the y intercept, which is given the symbol c.
- The y intercept is the point where the line passes through the y axis. The lines below have y intercepts of 5, 2 and -2.
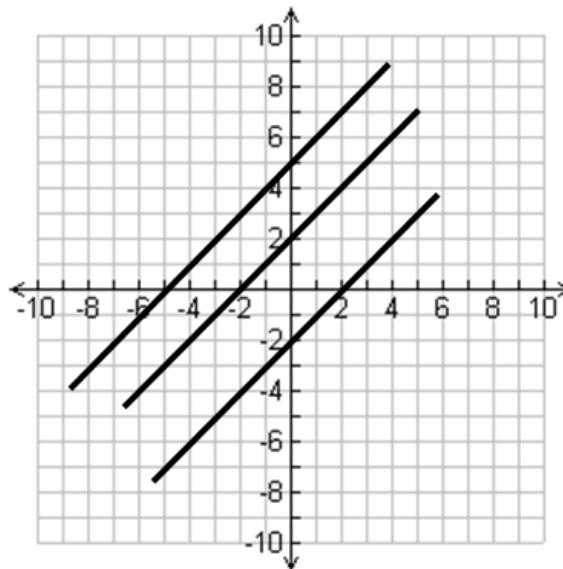
15. Your Turn:

Calculate the gradient and y intercept for the two lines, a and b, below.
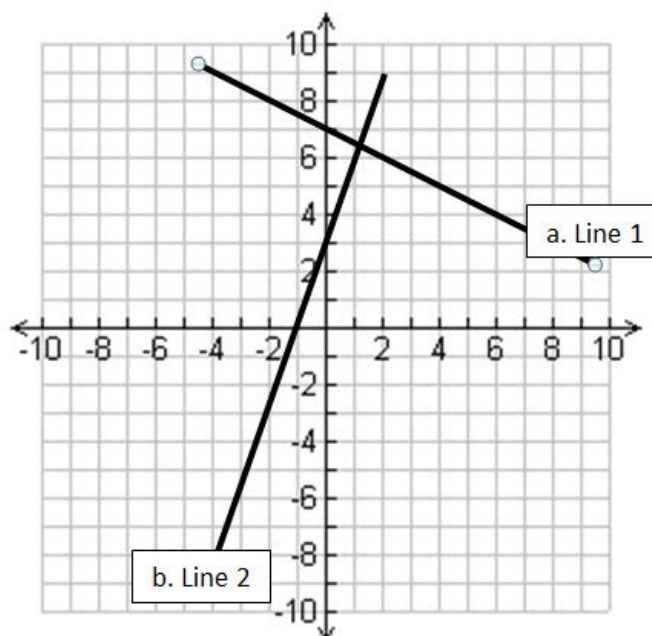
For further explanation click this link for Khan Academy:
"Slope and y-intercept intuition"
https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:linear-equations-graphs

17. Linear Equation

Now that we can calculate the gradient and y intercept, we can show the equation for a linear graph. All linear graphs have the following format:

$$y = mx + c$$

- So the equation: $y = 2x + 3$ has a positive gradient of 2 and has a y intercept of 3.
- From the 'your turn' problem from the previous page, we can now show that the two lines have the equations:
    - $y = 3x + 3$
    - $y = -0.5x + 7$
- Now that we have the equation we can use it to predict points.
    - For the line $y = 3x + 3$, when x equals 12, we substitute x for 12, to get $y = 3 \times 12 + 3$ $\therefore y$ must equal 39.

16. Your Turn:

Write the equation for each of the lines below and calculate the y value when $x = 20$.

a)

b)
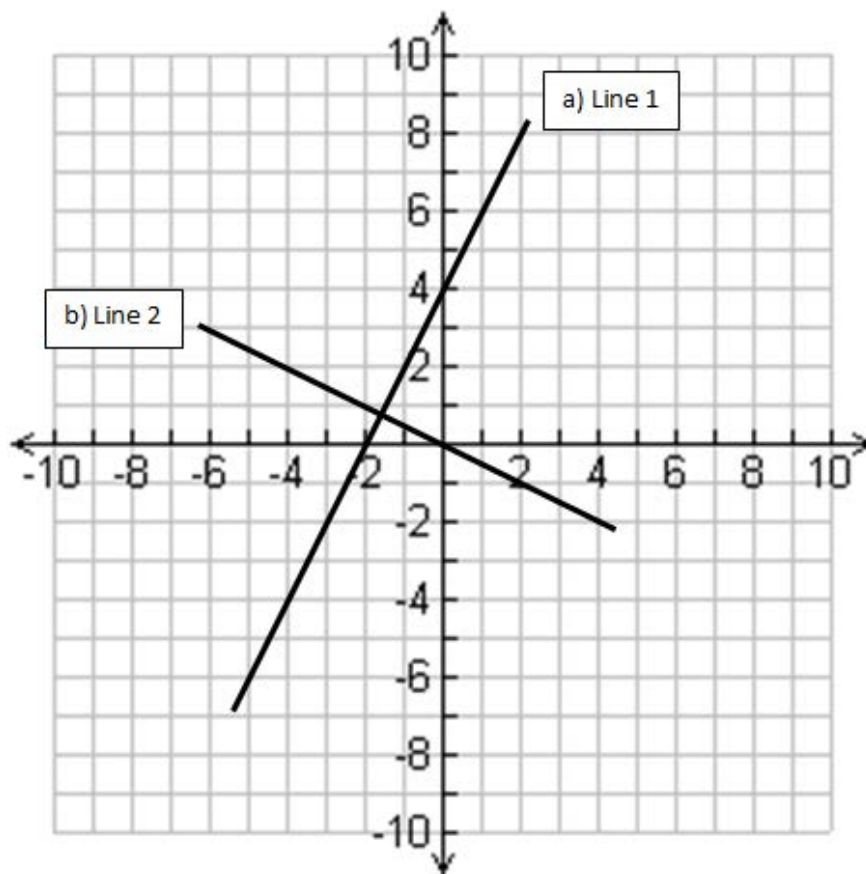
18. Graphing Equations

So far our focus has been on linear graphs, but some equations produce curved graphs.

Example Problems: Show the following lines on a graph and comment on the shape. Create a value table first.

1. $y = 2x$
2. $y = x^2$
3. $y = \frac{1}{x}$

1. $y = 2x$

X value	-3	-2	-1	0	1	2	3
Y value	-6	-4	-2	0	2	4	6
Coordinate	(-3, -6)	(-2, -4)	(-1, -2)	(0, 0)	(1, 2)	(2, 4)	(3, 6)

2. $y = x^2$

X value	-3	-2	-1	0	1	2	3
Y value	9	4	1	0	1	4	9
Coordinate	(-3, 9)	(-2, 4)	(-1, 1)	(0, 0)	(1, 1)	(2, 4)	(3, 9)

3. $y = \frac{1}{x}$

X value	-3	-2	-1	0	1	2	3
Y value	-0.33	-0.5	-1	error	1	.5	.33
Coordinate	(-3, -0.33)	(-2, -0.5)	(-1, -1)		(1, 1)	(2, .5)	(3, .33)

Step 2: Plot the graph

y=2x

$y = x^2$

$y = \frac{1}{x}$
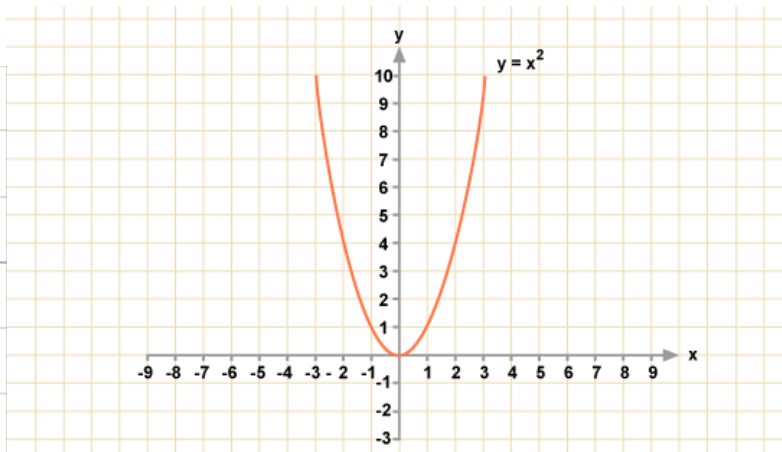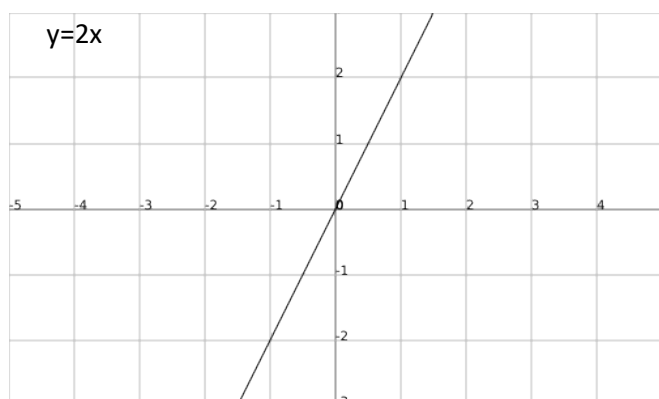
Step 3: Comment on the shape

1. Positive Linear
2. Non Linear (Parabola)
3. Non Linear (does not touch zero)
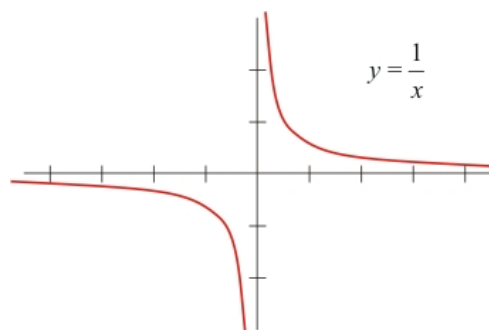
Common curve shapes:

x^2	$-x^2$	x^3	x^4	e^x	$\ln x$

17. Your Turn:

On the graph below, plot the following three equations and comment on the shape.

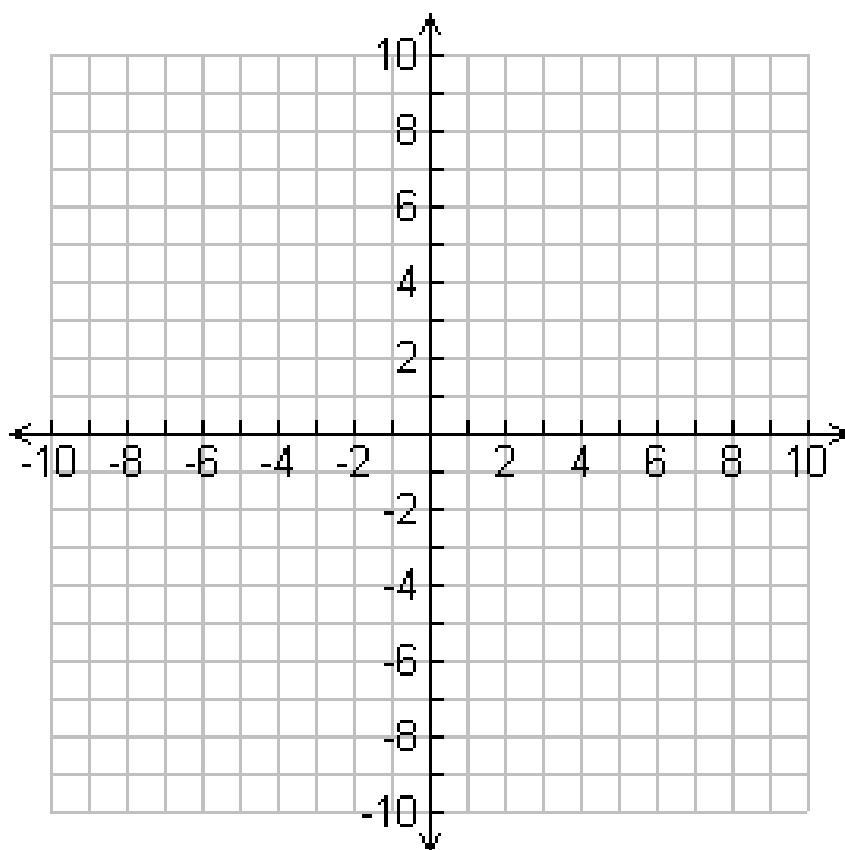 a) $y = 3x - 2$
  b) $y = x^2 + 1$
  c) $y = -x^2 + 4$

a)

X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							

b)

X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							

c)

X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							

19. The Gradient of a Curve

In the previous section when calculating the gradient of a straight line, we used the formula $m = \frac{\Delta y}{\Delta x}$. Hence, at any point on the line, the gradient remains the same. The example below left, always has a gradient of $+0.6$.

A curve is different; the slope of a curve will vary at different points. One way to measure the slope of a curve at any given point is to draw a **tangent**. The tangent is a straight line that will just touch the curve at the point to be measured. An example is shown in the image below right; the tangent drawn shows the gradient at that point. If the point was moved either left or right, then the angle of the tangent changes.

- The gradient of the tangent at that point equals the gradient of the curve at that point.
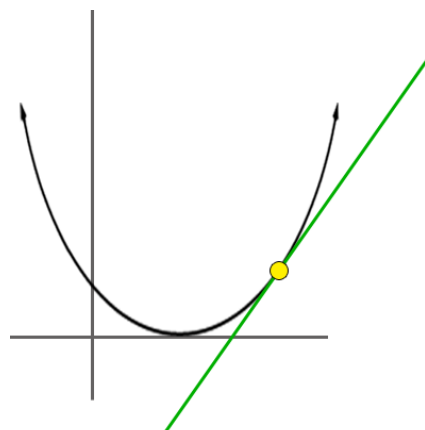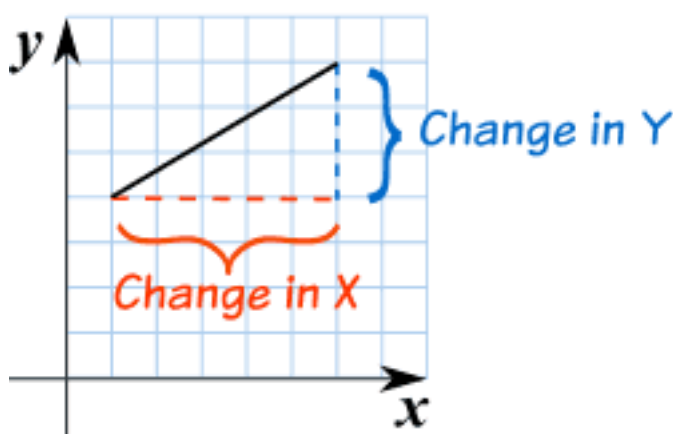
Example problem: Let's use the information from 18. b. and the plot you have drawn.

1. Plot a graph for $y = x^2 + 1$, for values of x between -3 and 3; hence, we created a table of values:

X value	-3	-2	-1	0	1	2	3
Y value	10	5	2	1	2	5	10
Coordinate	-3,10	-2,5	-1,2	0,1	1,2	2,5	3,10

2. Draw in a tangent A at (1,2)

3. Calculate the gradient of the tangent (drawn by eye at this stage)and find the gradient of the curve at A.

Thus we select two points on the tangent which are $(-1, -2)$ and $(-3, -6)$

Now calculate the gradient of the tangent: $m = \frac{\Delta y}{\Delta x}$

which is: $gradient = \frac{difference\ in\ y\ coordinates}{difference\ in\ x\ coordinates}$

$m = \frac{-2 - (-6) = 4}{-1 - (-3) = 2} = 2 \therefore$ the gradient of the tangent is 2 and thus the gradient of the curve at point A(1,2) is 2.

18. Your Turn:

Using the information above, calculate the gradient of a tangent at B $(-2,5)$.
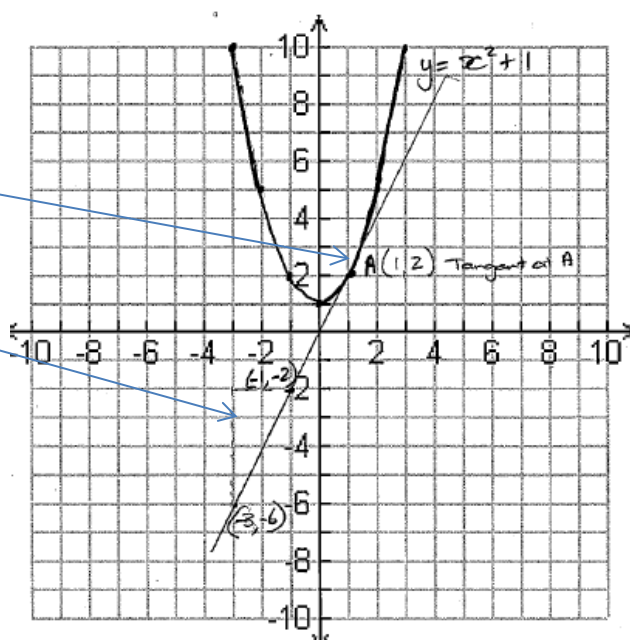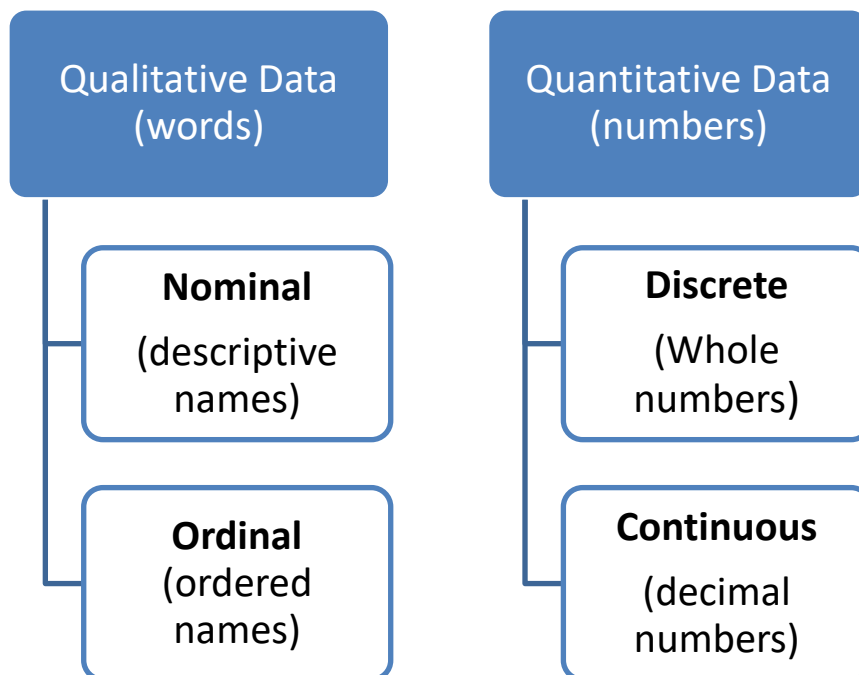
20. Types of Data

Statistics is focused on the analysis of data. The type of data determines how statistics are used. In the broadest sense, data falls into two categories: Qualitative and Quantitative.

Qualitative data:

- Is based on observations which focus on a discrete "quality"
- These observations form categories.
- Qualitative data is further split into two groups
  - Nominal Data: Uses names for each category. E.g. Red, Blue, green etc.
  - Ordinal data: Uses names which have an order. E.g. Distinction, Credit, Pass

Quantitative data:

- Is based on observations which focus on the "quantity" or how many.
- These observations are numerical.
- Quantitative data is further split into two groups
  - Discrete Data: Often uses whole numbers, no in-between values. E.g. Shoe size
  - Continuous Data: Often decimal numbers, with a large range of values. E.g. Height
- Note: If in doubt ask if a decimal number e.g. 9.665 would make sense. You can't buy a size 9.665 shoe (discrete) but the width of your shoe could be 9.665cm wide.

Qualitative Data (words)		Quantitative Data (numbers)	
Nominal (descriptive names)		**Discrete** (Whole numbers)	
Ordinal (ordered names)		**Continuous** (decimal numbers)	

21. Measures of Central Tendency

When examining a collection of data, it is most common to find a measure of the "middle" value. The three common measures are:

- o **Mean:** The most commonly used measure. Often referred to as the average. It is calculated by adding up all the data and dividing by how many pieces of data (observations) you had.
- o The formula for mean is: $\bar{x} = \frac{\Sigma x}{n}$ $\bar{x}$ is called "x bar" and represents mean.

  Σ is called "sigma" and means sum or add up

  x is the individual observation

  n is the total number of observations
- o **Median:** The middle number when the data is arranged in order.
- o **Mode:** The most common number (a data set can have more than 1 mode e.g. bimodal has 2)

Example Problem:

The following marks (out of 10) were collated from a recent math test:

7	5	9	10	5	6	7	4	7	6	7	10

Step 1: Arrange in order

4	5	5	6	6	7	7	7	7	9	10	10

Step 2: Mean

$$\bar{x} = \frac{\Sigma x}{n}$$
$$\bar{x} = \frac{4+5+5+6+6+7+7+7+7+9+10+10}{12}$$
$$\bar{x} = \frac{83}{12}$$
$$\bar{x} = 6.9$$

Step 3: Median

4	5	5	6	6	7	7	7	7	9	10	10

Counting in from either side you can see the middle is somewhere between 7 and 7. The Median is 7

Step 4: Mode is 7

19. Your Turn: Calculate the mean, median and mode of the following data

2	5	8	4	9	3	4	7	4	8	3	1

22. Central Tendency Considerations

- Measures of central tendency are very powerful tools when comparing data, but they must be used when appropriate. The type of data determines which method should be used.
- For qualitative data, the most meaningful measure of central tendency is mode. For quantitative data, the mean and median are the most meaningful measure of central tendency.

- **Nominal Data:** e.g.   Blue   Blue   Blue   Brown   Green   Brown   Grey

  Mean: Is meaningless
  Median: Is meaningless
  Mode: Is valid; Blue is the most common colour.

- **Ordinal Data**: e.g.   Credit   Credit   Distinction   Pass   Pass   Fail

  Mean is meaningless
  Median: Is partially valid; middle score is between a pass and a credit
  Mode: Is valid; the most common grade was pass and credit.

- **Discrete Data:** e.g.   8   8   6   11   9   12   10

  Mean: Is Valid. 9.14 is the mean.
  Median: Is valid. 9 is the middle number.
  Mode: Is valid. 8 is the most common.

- **Continuous Data**: e.g.   2.5   2.7   3.4   5.2   4.6   1.9
  Mean: Is valid. 3.38 is the mean
  Median: Is valid. It is between 2.7 and 3.4. The middle point is 3.05
  Mode: Is valid. They are all equally common. This is not unusual for this type of data.

- **Outliers** are common in real world data. Certain values in a group of data that don't "fit" into the data supplied. Outliers may be due to human error or they may be observations of actual phenomena. It is important to recognise this.
  - Data set 1: Marks out of 100.   94   87   75   98   83   92   82
  - Data set 2: Marks out of 100.   94   87   75   9.8   83   92   82

  Can you spot the outlier in these data sets?

  For Data set 1: Mean = 87.28   Median = 87
  For Data set 2: Mean = 74.68   Median = 83

  Note that the Mean was more affected by an outlier than the median. Mean, as a measure of central tendency, is most effective when the data is "normal". The observation of 9.8 in the second data set is an outlier. If it is due to a typing error the erroneous value should be ignored. If a student actually scored 9.8 out of 100 it cannot be ignored.

Watch this short Khan Academy video for further explanation:
"Comparing means of distributions"
https://www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/v/comparing-distribution-means

23. Measures of Spread

Data can be very closely grouped together, or it might be spread far apart and still have the same mean.

Data set 1:	1	10	10	1	2	9	$\bar{x} = 5.5$
Data set 2:	5	6	6	5	6	5	$\bar{x} = 5.5$

Measures of spread are used to determine this difference in data. The smaller the value the more closely the data is clustered around the middle.

Range: Most basic measure of spread. Subtract the largest value from the smallest value.

E.g. 2 6   7   8   7   6   9   5   Range is 9 – 2 = 7

Interquartile Range: (IQR)This measure is linked with Median. If you calculate the median of a data set you actually calculate the half-way point. Interquartile range is based on subtracting the ¼ point from the ¾ point. This is not as straightforward as Range, but can be demonstrated in the example below:

4   5   **5**   **6**   6   **7**   **7**   7   **7**   **9**   10   10
          25%   50%   75%

Median is 7.

1st quartile is 5.5.

3rd quartile is 8

Interquartile Range is 8 - 5.5 = 2.5

Standard Deviation: (σ) This measure is linked with the mean value of a data set. This calculates how far, on average, each value is from the mean. The formula for most cases is:

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$$

Calculating standard deviation manually is a time consuming process, and as such, it is most common to use a calculator or computer to make this calculation. An example of a manual standard deviation calculation is provided below:

Data: 7   9   5   6   3 $\bar{x} = \frac{30}{5}$ $= 6$

n = 5

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$\sigma = \sqrt{\frac{(7-6)^2 + (9-6)^2 + (5-6)^2 + (6-6)^2 + (3-6)^2}{5-1}}$$

$$\sigma = \sqrt{\frac{(1)^2 + (3)^2 + (-1)^2 + (0)^2 + (-3)^2}{4}}$$

$$\sigma = \sqrt{\frac{1 + 9 + 1 + 0 + 9}{4}}$$

$$\sigma = \sqrt{\frac{20}{4}}$$

$$\sigma = \sqrt{5}$$

$$\sigma = 2.24$$

Watch this short Khan Academy video for further explanation:
"Range, variance and standard deviation as measures of dispersion"
https://www.khanacademy.org/math/probability/descriptive-statistics/variance_std_deviation/v/range-variance-and-standard-deviation-as-measures-of-dispersion

24. Bias and Skewed Data

- **Bias** is the most important, and least well executed, part of statistics. Bias is either the accidental or deliberate manipulation of data so that the statistics of a sample are not representative of the population. The best mechanism to prevent bias is randomisation.

An example:

I want to determine if Australia wants to become a republic rather than a monarchy. I have the following options:

1. Ask 10 of my friends in my science lecture.
2. Ask all 22 million Australians to vote on the question.
3. Send 1000 surveys to random address from the Cairns phone book
4. Put the question on twitter and tally the result after 5 days.

Option 1 is biased because it is a very small sample and my friends are not random.
Option 2 has the least bias but is incredibly expensive – this approach is called a census.
Option 3 is biased because my sample is only from Cairns and most people may not reply
Option 4 has the potential to be biased – need to know about the demographics of the respondents.

It is important to consider any sources of bias when viewing data.

- **Skew** is a description of the distribution of the data. A normal distribution is data which is symmetrical when graphed; all three measures of central tendency overlap. In a skewed distribution, the graph is not symmetrical. Skewed data is often referred to as "having a tail". A distribution may be positively skewed or negatively skewed. The mean, median and mode are then spread accordingly, as is represented in the graphs below:
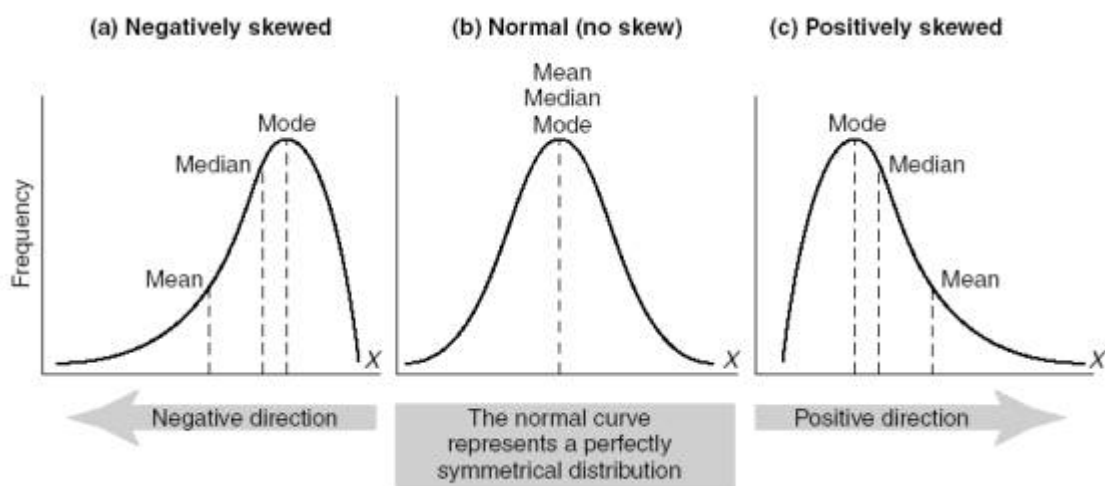
Examples of normal and skewed distributions

25. Graph Basics

- The purpose of graphs is to display data in a simple to understand format. Graphs compare variables or groups of data.
- A graph has two axes (axes = plural; axis = singular) The X axis is horizontal. The Y axis is vertical

- The following checklist should be used when graphing:
    - Graph Title
    - X axis Title
    - X axis Units
    - Y axis Title
    - Y axis units
    - Axes are spaced evenly and do not go up in 3's
    - The Y axis begins at zero
    - If hand drawn, the graph fills at least ¾ of the graph paper.
   - If multiple variables are shown, a key must be used

Graph of Mass vs. Height
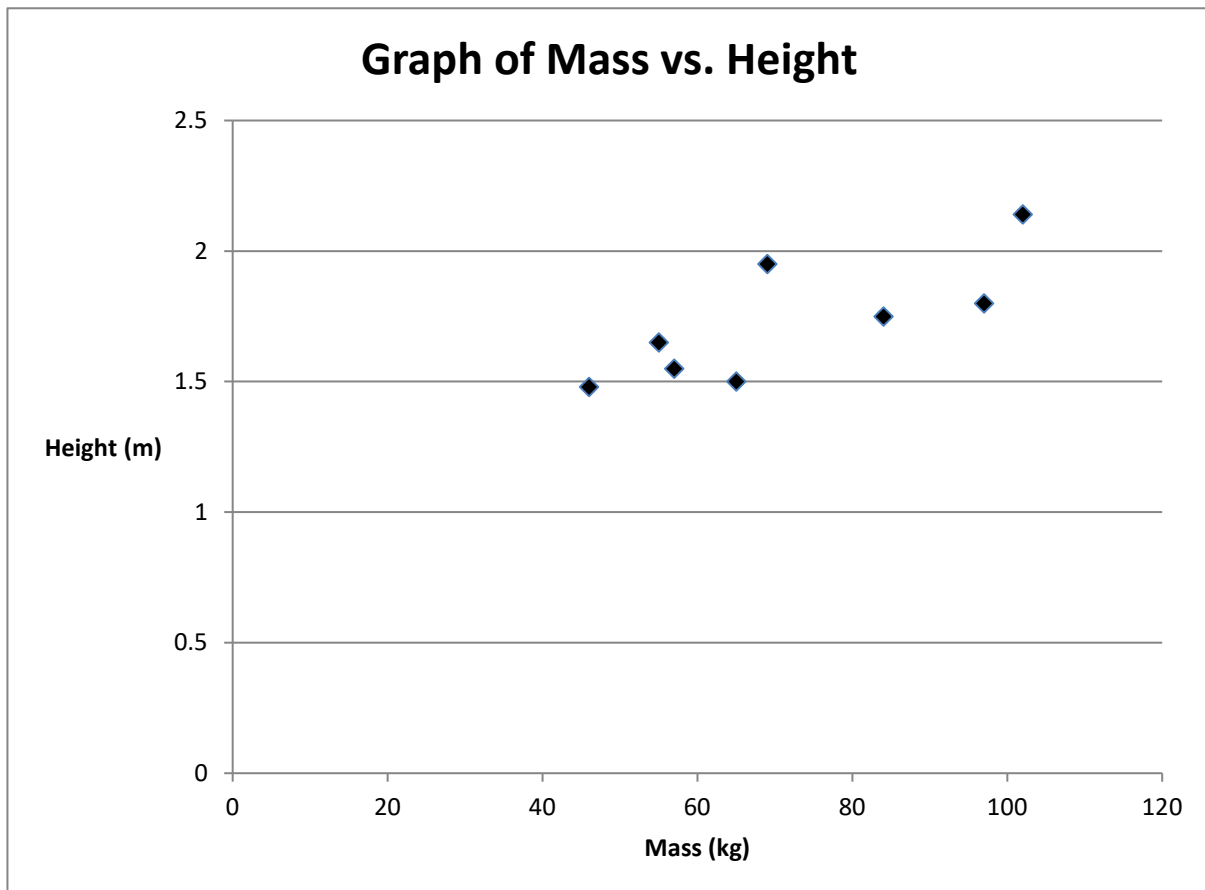
- When graphs are marked, it is common that the actual data plotted is only worth ¼ of the graphs total marks.
- If possible, computer generated graphs are preferable to hand drawn graphs.

26. Types of Graph

- The choice of graph is determined by the types of data used.

Bar Graph: Used to display qualitative Data.

Useful for frequency data.

Often have words along the x axis.

Must have gaps between the bars.

Birthdays of Students by Month
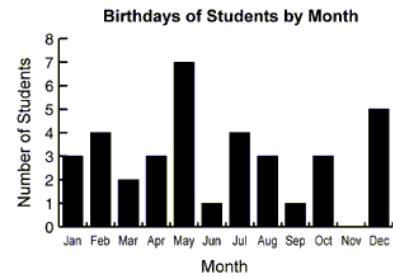
Pie Chart: Used to display qualitative data.

Are easy to read but provide limited information.

Used often to compare parts to the total (as percentage %).

Pie Preferences
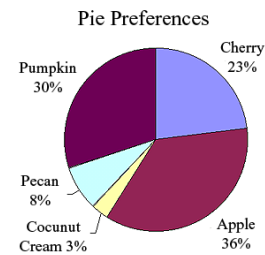
Histogram: Used to display quantitative data.

Has numbers along the x axis.

Must have no gaps between the bars.
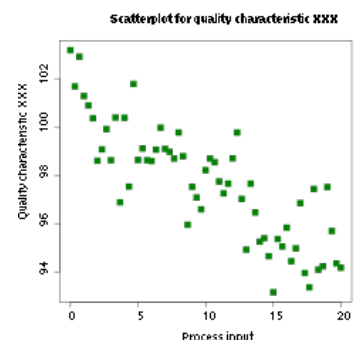

Heights of Black Cherry Trees
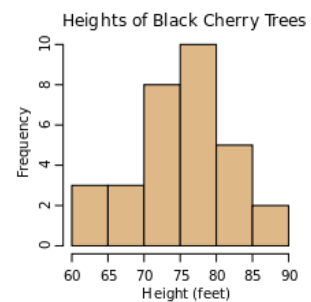
Box and Whiskers: Used to display quantitative data.

Displays the median and interquartile

Excellent for skewed data

Figure 2. Carl's and Angela's box and whisker plots
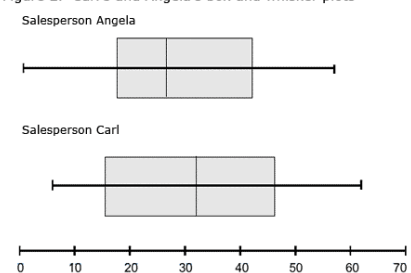
Scatterplot: Used to compare two quantitative variables

Designed to show correlation

Most commonly used in science

Scatterplot for quality characteristic XXX

27. Box plot

- The box and whiskers plot is one of the most time consuming graphs and most difficult to draw in excel.
- It display 5 values :
  - Lowest Value
  - 1st Quartile
  - Median
  - 3rd Quartile
  - Highest Value
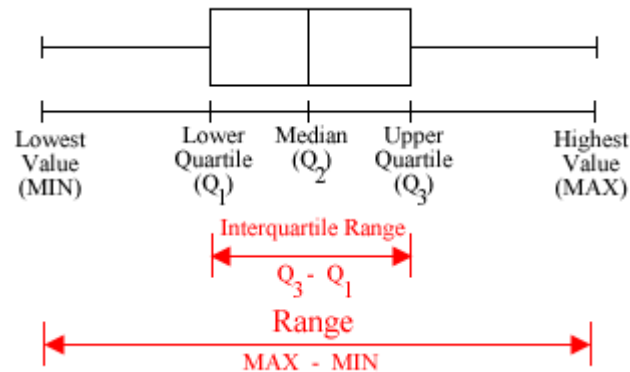
Example Problem:

Draw a Box Plot of the following data: (Physics Test: marks out of 20)

| 2 | 7 | 5 | 9 | 3 | 6 | 14 | 8 | 4 | 9 | 2 | 5 |

Step 1: Arrange in order:

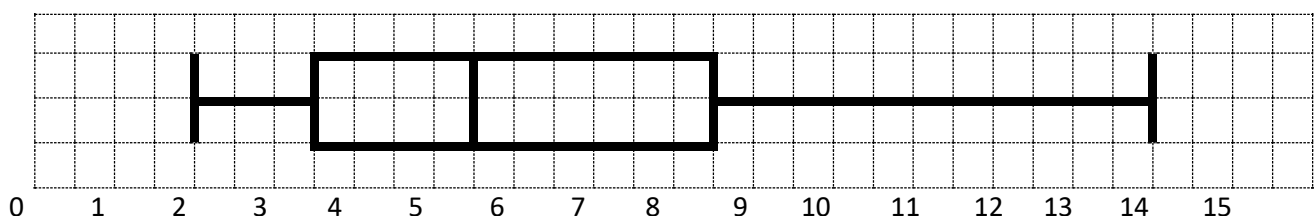| 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 | 9 | 9 | 14 |

Step 2: Calculate the following:

- Lowest Value: 2
- 1st Quartile: 3.5 (half way between 3 and 4)
- Median   5.5 (half way between 5 and 6)
- 3rd Quartile   8.5 (half way between 8 and 9)
- Highest value   14

Step 3: Draw in the 5 lines and draw a box

Physics Test Results

Marks out of 20

Step 4: Add titles and labels

28. Scatter Plot

- Scatterplots are only used for two continuous variables. E.g. Height vs Mass.
- A scatterplot is designed to find relationships between the two variables.
- The most common relationships are linear. A relationship is evident when a diagonal line can be formed when the data is graphed.
- Relationships can be:
    - positive (↗) or negative (↘)
    - Strong (really straight line) or weak (line requires some imagination)
    - No relationship (No obvious line or the line is perfectly vertical or horizontal)
    - Non-Linear relationship (forms a curved line)
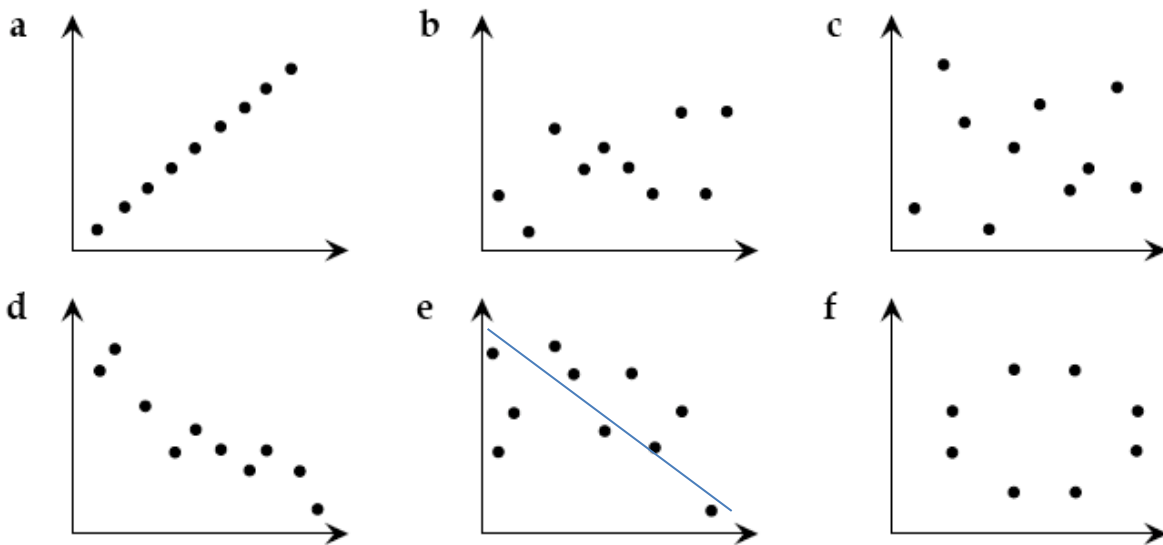
- Looking at the graphs above:
    a. Is a Very Strong Positive Relationship
    b. Is a Weak Positive or No Relationship
    c. Is no relationship
    d. Is a Strong Negative Relationship
    e. Is a Weak Negative Relationship
    f. Is a Non Linear Relationship
- To display the relationship, we use a "line of best fit" When the graph is hand drawn, it is a line which tries to have as many points above the line as there are below the line (shown on graph e).
- Using Microsoft Excel, it is possible to have a "line of best fit" drawn using the trend line menu.
- The quality of the relationship can be measured in excel by displaying the r^2 value. It is calculated in a similar way to standard deviation, and gives a value between 0 and 1. A straight line = 1, or -1. No relationship = 0.

29. Answers

1. What is Algebra (page 2)

   a. 1   b. 22   c. 10   d. 12

2. Glossary (page 3)

   a. 27   b. 6   c. 4   d. no (this expression is only one side of an equation which has an equals sign to indicate equivalence of both sides.)   e. x is a variable and a factor of $4x$; 9 is a constant; + is an operator.

3. Multiplication Properties (page 4)

   a. $3 \times 2x \ or \ 2 \times 3x$; $(6x$ simplified)   b. $(2 \times 4)x$ c. $(8 \times 2) + (8x) \ or (16 + 8x)$

4. Collecting Like Terms (page 5)

   a. $3x + 2y - x = 2x + 2y$
   b. $2x^2 - 3x^3 - x^2 + 2x = x^2 - 3x^3 + 2x$
   c. $3m + 2n + 3n - m - 7 = 2m + 5n - 7$
   d. $4(x + 7) + 3(2x - 2) = 4x + 28 + 6x - 6$
$$= 10x + 22$$
   e. $3(m + 2n) + 4(2m + n) = 3m + 6n + 8m + 4n$
$$= 11m + 10n$$
   f. $\frac{x}{3} + \frac{x}{4} = \frac{(4x+3x)}{12} = \frac{7x}{12}$

5A. Simplifying Expressions: Using expansion (page 6)

   a. $4x + 3x, or \ 7x$ b. $3x - x, or \ 2x$ c. $8x + 6x, or \ 14x$

5B. Simplifying Expressions: Using expansion (continued). (page 6)

   d. 16   e. 18   f. 120   g. 4   h. 2   i. 3

6. Simplifying Equations with Exponents (page 7)

   a. $20x$ b. $11xy$ c. x^2 d. $5x^2+10x$

7. Solving Equations (page 9)

   a. $x = \frac{y}{3}$ f. $x = \frac{y + 26}{13}$ $y + 3t - a = tx \ \therefore x = \frac{y+3t-a}{t}$
   b. $x = \frac{1}{y}$ g. $y = x + 1$ and so   i. $x = 7$
                           $x = y - 1$ j. $x = 162$
   c. $x = \frac{5+y}{7}$ h. $y = a + tx - 3t$ and so   k. $y = 12$
   d. $x = 2y + 14$ $y + 3t = a + tx$ then   l. $x = 13$
   e. $x = \frac{1}{2y}$ m. $x = 154$

8. Multiples and Factors (page 10)

   a. 1, 2, 7, 14   b. 1, 2, 67, 134   c. 1, 2, 4, 7, 8, 14, 28, 56   d. 1, 3, 9, 27   e. 1, 2, 61, 122
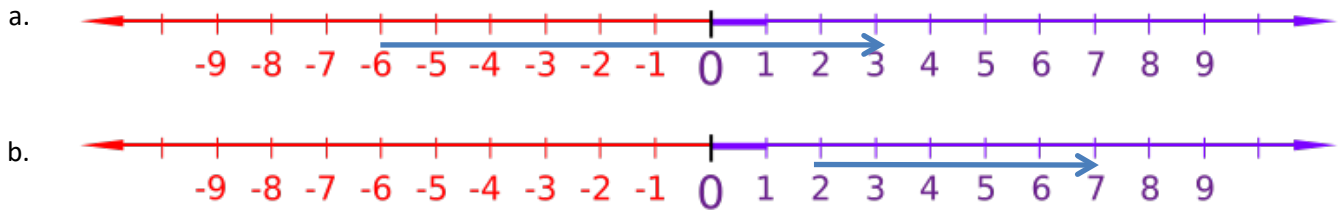
9. Expanding and Factorising (page 11)

   a. $6xy^2 + 14y^3 + 4y^2$ c. $3x(1 + 2x - 3)$
   b. $-3x + (-2)$ d. $6x(4 + 7y - 10x^2)$

10. Function Notation (page 12)

   a. f(4) = 33; f(0) = 9   b. f(6) = 37; f(-6) = 35

11. The Number Line (page 13)

   a.

   b.

12. The Cartesian Plane (page 14)

   a. quadrant II $(-3, 5)$ b. quadrant III $(-3, -5)$ c. quadrant IV $(3, -5)$

13. Coordinates (page 15)
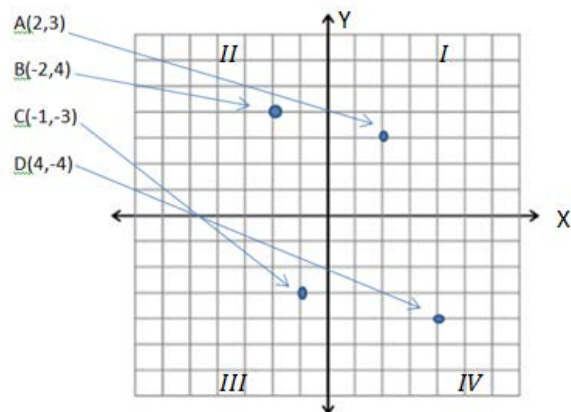
14. Gradient (page 17)

   a. $-\dfrac{7}{4} = -1\dfrac{3}{4} = -1.75$

15. Intercept (page 18)

   a. Line 1: $m = -0.5, c = 7$ b. Line 2: $m = 3, c = 3$

16. Linear Equation (page 19)

   a. Line 1: $y = 2x + 4$ $\therefore y = 44$ $when$ $x = 20$
   b. Line 2: $y = -\dfrac{1}{2}x + 0$ $\therefore y = -10$ $when$ $x = 20$

17. Graphing Equations (page 21)

   a. Positive Linear   b. U Shaped   c. ∩ Shaped

18. The Gradient of a Curve (page 22)

   a. -4

19. Measures of Central Tendency (page 24)

   Mean: 4.83   Median: 4   Mode: 4

30. Glossary

Algebra The part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations

Associative property It doesn't matter how you group the numbers when you multiply or add. (In other words, it doesn't matter which you calculate first.), e.g. $(6 + 3) + 4 = 6 + (3 + 4)$, $(2 \times 4) \times 3 = 2 \times (4 \times 3)$
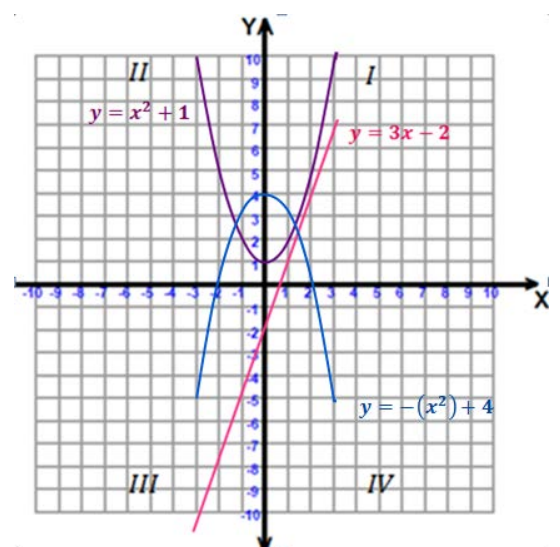
Bar graph A graph using rectangular bars to show how large each value is (the bars can be horizontal or vertical); it is often used to display qualitative data and must have gaps between the bars

Bias The accidental or deliberate manipulation of data so that the statistics of a sample are not representative of the population

Box and whiskers plot A graph showing the distribution of a set of data along a number line, dividing the data into four parts using the median and quartiles

Cartesian plane A plane which has a horizontal line (x axis) and a vertical line (y axis)

Composite number Any whole number that has more than two factors (opposite of a prime number), e.g. 4, 6, 8, …

Coefficient A number that is partnered with a variable (coefficients of 1 are not shown); between the coefficient and the variable is a multiplication, e.g. in 4x, the coefficient is 4

Common factor A factor that two or more numbers have in common, e.g. 3 is a common factor of 12 and 15

Common multiple A multiple which two or more numbers have in common, e.g. 3 and 5 have multiples in common: 15, 30, 45…

Commutative property You can swap numbers around and still get the same answer when you add or when you multiply, e.g. $3 + 6 = 6 + 3$, $2 \times 4 = 4 \times 2$

Constant A number on its own, or sometimes a letter such as a, b or x that stands for a fixed number, e.g. in $x + 5 = 9$, 5 and 9 are constants

Continuous data A type of quantitative data with a large range of values; it often uses decimal numbers, e.g. height

Coordinates A pair of numbers (the x coordinate and the y coordinate) which give you the location of a point on the Cartesian plane, e.g. (2;3) (the x value is always written first)

Correlation When two sets of data or variables are linked together; the correlation is positive when the values increase together and negative when one value increases as the other decreases

Discrete data A type of quantitative data that doesn't have in-between values and often uses whole numbers, e.g. number of pets

Distributive Multiplying a number by a group of numbers added together is the same as doing each multiplication separately, e.g. $3 \times (2 + 4) = 3 \times 2 + 3 \times 4$

Equation A mathematical sentence; it contains an equal sign meaning that both sides are equivalent, e.g. $2x+3 = 17$

Expanding Removing the brackets in an expression using the distributive law, e.g. 4(a + 3) = 4 x a 4 x 3 = 4a + 12

Expression Numbers, symbols and operators (such as + and ×) grouped together that show the value of something, e.g. 2 x 3

Factor A whole number that can be multiplied a certain number of times to reach a given number, e.g. 3 is a factor of 15 because it can be multiplied by 5 to get 15

Factorising Writing a number or algebraic expression as a product (the opposite of expanding), e.g. 2y + 6 = 2(y + 3)

Factor tree A tree diagram that shows the prime factors of a number

Function A special relationship between values: each of its input values gives back exactly one output value; it is often written as "f(x) = …" where x is the value you give it

Gradient (also called slope) How steep a line is; it has the symbol "m" and can be calculated by dividing the difference in y coordinates by the difference in x coordinates (rise over run)

Highest common factor (HCF) The highest factor that two or more numbers have in common, e.g. the HCF of 12 and 30 is 6, because 1, 2, 3 and 6 are factors of both 12 and 30, and 6 is the highest

Histogram A graph representing data which is grouped into ranges (such as "40 to 49"), and then plotted as bars; often used to display quantitative data and must have no gaps between the bars

Intercept The point at which a line crosses the y-axis (y intercept) or x-axis (x intercept) of the Cartesian plane

Interquartile range The difference between the first quartile and third quartile of a set of data; it is one way to describe the spread of a set of data

Like terms Terms whose variables (and their exponents such as the 2 in x^2) are the same; In other words, terms that are "like" each other, e.g. 7x, x, and -2x are like terms because the variables are all x

Linear equation An equation that makes a straight line when it is graphed; often written in the form y = mx + c

Lowest common multiple (LCM) The lowest multiple that two or more numbers have in common, e.g. 15 is the LCM of 3 and 5

Mean A calculated "central" value of a set of numbers (often called average); to calculate, add up all the numbers, then divide by how many numbers there are

Median The middle number in a sorted list of numbers

Mode The number that appears most often in a set of numbers

Multiple The result of multiplying a number by an integer (not a fraction), e.g. 12 is a multiple of 3, because 3 × 4 = 12

Nominal data A type of qualitative data that uses names for each category, e.g. red, blue, green etc.

Number line A visual representation of all the positive and negative numbers

Operator Symbols used between numbers to indicate a task or relationship, e.g. +, -, x, =

Ordinal data A type of qualitative data that uses names that have a logical order, e.g. distinction, credit, pass etc.

Origin The point of intersection of the x-axis and the y-axis in a Cartesian plane (0;0)

Outlier A value that "lies outside" (is much smaller or larger than) most of the other values in a set of data

Parabola A curve where any point is at an equal distance from (1) a fixed point and (2) a fixed straight line; if you kick a soccer ball (or shoot an arrow, fire a missile or throw a stone) it will arc up into the air and come down again following the path of a parabola

Pie chart A circular chart divided into sectors, each sector shows the relative size of each value

Prime factor A factor that is also a prime number

Prime number Any whole number greater than zero that has exactly two factors - itself and one, e.g. 2, 3, 5, 7, 11, …

Proper factor All factors of a number apart from the number itself

Tangent A line that just touches a curve at one point, without cutting across it

Term Parts of an expression separated by operators; it can be a single number, or a variable, or numbers and variables multiplied together e.g. in 4x - 7 = 5, there are three terms: 4x, 7, and 5

Quadrant Any of the 4 equal areas made by dividing a Cartesian plane by its x and y axis; quadrants are usually numbered I, II, III and IV (anticlockwise starting from the top right)

Qualitative data Information that focuses on a discrete "quality", e.g. traits and attributes that are not numeric such as colour or an opinion

Quantitative data Information that can be counted or measured and focuses on "quantity", e.g. a count or percentage

Randomisation Making something random to prevent bias; this means generating a random permutation of a sequence, e.g. electing a random sample of a population, or allocating experimental units via random assignment to a treatment or control group

Range The difference between the lowest and highest values in a set of data, e.g. in {4, 6, 9, 3, 7}, the lowest value is 3, and the highest is 9, so the range is 9-3 = 6.

Scatterplot A graph of plotted points that show the relationship between two sets of data; often used to compare two quantitative variables and show a correlation

Skewed data The graph of skewed data is not symmetrical and is often referred to as "having a tail"; data can be positively skewed (the long tail is on the positive/right side of the peak) or negatively skewed (the long tail is on the negative/left side of the peak)

Standard deviation The Standard Deviation is a measure of how spread out the numbers are; it is the square root of the variance, and the variance is the average of the squared differences from the mean

Variable A letter that represents an unknown number; most commonly "x", but it can be any symbol

31. Resources

BIDMAS: http://www.educationquizzes.com/gcse/maths/bidmas-f/

Commutative property:

http://www.mathematicsdictionary.com/english/vmd/full/c/vepropertyofmultiplication.htm

http://www.purplemath.com/modules/numbprop2.htm

Like Terms: http://www.freemathhelp.com/combining-like-terms.html

Just math: http://patrickjmt.com/