

# Mathematics for Earth Science

The module covers concepts such as:

- Maths refresher
- Fractions, Percentage and Ratios
- Unit conversions
- Calculating large and small numbers
- Logarithms
- Trigonometry
- Linear relationships



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# Mathematics for Earth Science

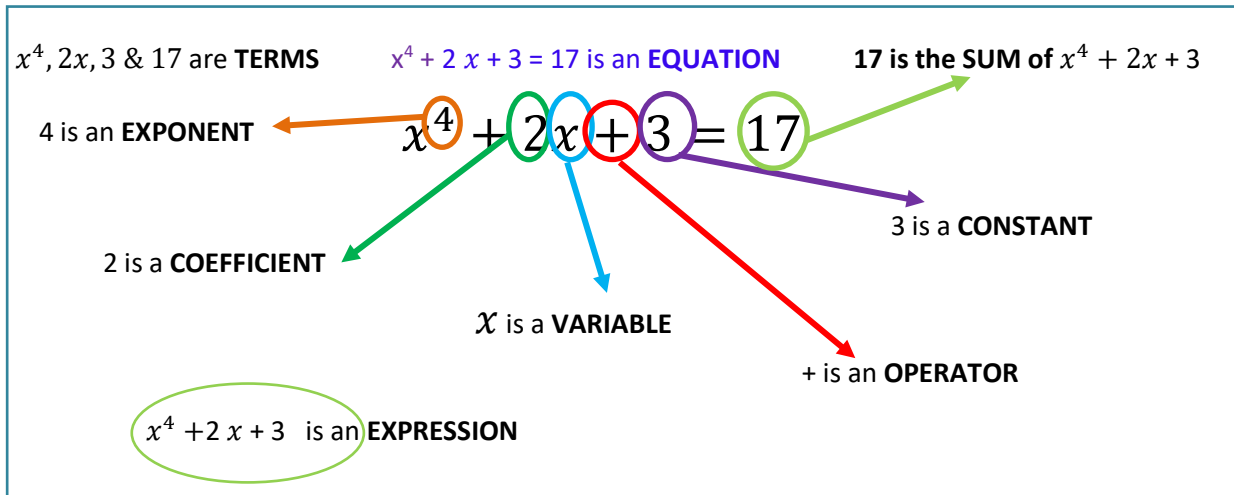
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# 1. Terms and Operations

## Glossary



- Equation:** A mathematical sentence containing an equal sign. The equal sign demands that the expressions on either side are balanced and equal.
- Expression:** An algebraic expression involves numbers, operation signs, brackets/parenthesis and variables that substitute numbers but does not include an equal sign.
- Operator:** The operation (+, −, ×, ÷) which separates the terms.
- Term:** Parts of an expression separated by operators which could be a number, variable or product of numbers and variables. Eg.  $2x$ ,  $3$  &  $17$
- Variable:** A letter which represents an unknown number. Most common is  $x$ , but can be any symbol.
- Constant:** Terms that contain only numbers that always have the same value.
- Coefficient:** A number that is partnered with a variable. The term  $2x$  is a coefficient with variable. Between the coefficient and variable is a multiplication. Coefficients of 1 are not shown.
- Exponent:** A value or base that is multiplied by itself a certain number of times. I.e.  $x^4$  represents  $x \times x \times x \times x$  or the base value  $x$  multiplied by itself 4 times. Exponents are also known as Powers or Indices.

## In summary:

Variable:	$x$	Operator:	+
Constant:	$3$	Terms:	$3, 2x$ (a term with 2 factors) & $17$
Equation:	$2x + 3 = 17$	Left hand expression:	$2x + 3$
Coefficient:	$2$	Right hand expression	$17$ (which is the sum of the LHE)

The symbols we use between the numbers to indicate a task or relationships are the **operators**, and the table below provides a list of common operators. You may recall the phrase, 'doing an operation.

Symbol	Meaning
+	Add, Plus, Addition, Sum
−	Minus, Take away, Subtract, Difference
×	Times, Multiply, Product,
÷	Divide, Quotient
±	Plus and Minus
<i>a</i>	Absolute Value (ignore −ve sign)
=	Equal
≠	Not Equal
<	Less than
>	Greater than
≪	Much Less than
≫	Much More than
≈	Approximately equal to
≤	Less than or equal
≥	Greater than or equal
Δ	Delta
Σ	Sigma (Summation)

### Order of Operations

The **Order of Operations** is remembered using the mnemonic known as the BIDMAS or BOMDAS (**B**rackets, **I**ndices or **O**ther, **M**ultiplication/Division, and **A**ddition/**S**ubtraction).

<b>Brackets</b>	{{( )}}
<b>Indices or Other</b>	$x^2, \sin x, \ln x, etc$
<b>Multiplication or Division</b>	$\times$ or $\div$
<b>Addition or Subtraction</b>	+ or -

### The Rules:

1. Follow the order (BIMDAS, BOMDAS or BODMAS)
2. If two operations are of the same level, you work from left to right. E.g. ( $\times$  or  $\div$ ) or (+ or −)
3. If there are multiple brackets, work from the inside set of brackets outwards. {{( )}}

**Example Problems:**

1. Solve:  $5 + 7 \times 2 + 5^2 =$   
Step 1:  $5^2$  has the highest priority so:  $5 + 7 \times 2 + 25 =$   
Step 2:  $7 \times 2$  has the next priority so:  $5 + 14 + 25 =$   
Step 3: only addition left, thus left to right:  $19 + 25 = 44$   
 $\therefore 5 + 7 \times 2 + 5^2 = 44$

**Question 1:**

Here are some revision examples for practise:

- a.  $10 - 2 \times 5 + 1 =$
- b.  $10 \times 5 \div 2 - 3 =$
- c.  $12 \times 2 - 2 \times 7 =$
- d.  $48 \div 6 \times 2 - 4 =$
- e. *What is the missing operation symbol*  $18 \square 3 \times 2 + 2 = 14$

## 2. Fractions – addition, subtraction, multiplication and division

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*Adding and subtracting fractions* draws on the concept of equivalent fractions. The golden rule is that you can only **add and subtract fractions** if they have the **same denominator**, for example,

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}.$$

If two fractions do **not** have the same denominator, we must use equivalent fractions to find a “common denominator” before they can be added together.

In the example  $\frac{1}{4} + \frac{1}{2}$ , 4 is the lowest common denominator. Use the equivalent fractions concept to change  $\frac{1}{2}$  into  $\frac{2}{4}$  by multiplying both the numerator and denominator by two:  $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$

Now that the denominators are the same, addition or subtraction operations can be carried out.

In this example the lowest common denominator is found by multiplying 3 and 5, and then the numerators are multiplied by 5 and 3 respectively:

$$\frac{1}{3} + \frac{2}{5} = \frac{(1 \times 5) + (2 \times 3)}{(3 \times 5)} = \frac{5 + 6}{15} = \frac{11}{15}$$

Compared to addition and subtraction, *multiplication and division of fractions* is easy to do, but sometimes a challenge to understand how and why the procedure works mathematically. For example, imagine I have  $\frac{1}{2}$  of a pie and I want to share it between 2 people. Each person gets a quarter of the pie.

Mathematically, this example would be written as:  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ .

Remember that fractions and division are related; in this way, multiplying by a half is the same as dividing by two.

So  $\frac{1}{2}$  (two people to share) of  $\frac{1}{2}$  (the amount of pie) is  $\frac{1}{4}$  (the amount each person will get).

But what if the question was more challenging:  $\frac{2}{3} \times \frac{7}{16} = ?$  This problem is not as easy as splitting pies.

A mathematical strategy to use is: “Multiply the numerators then multiply the denominators”

$$\text{Therefore, } \frac{2}{3} \times \frac{7}{16} = \frac{(2 \times 7)}{(3 \times 16)} = \frac{14}{48} = \frac{7}{24}$$

An alternative method you may recall from school is to simplify each term first. Remember, ‘What we do to one side, we must do to the other.’

The first thing we do is look to see if there are any common multiples. For  $\frac{2}{3} \times \frac{7}{16} = ?$  we can see that 2 is a multiple of 16, which means that we can divide top and bottom by 2:

$$\frac{2 \div 2}{3} \times \frac{7}{16 \div 2} = \frac{1}{3} \times \frac{7}{8} = \frac{1 \times 7}{3 \times 8} = \frac{7}{24}$$

**Division of fractions** seems odd, but it is a simple concept:

You may recall the expression 'invert and multiply', which means we flip the divisor fraction (second term fraction). Hence,  $\div \frac{1}{2}$  is the same as  $\times \frac{2}{1}$

This 'flipped' fraction is referred to as the **reciprocal of the original fraction**.

Therefore,  $\frac{2}{3} \div \frac{1}{2}$  is the same as  $\frac{2}{3} \times \frac{2}{1} = \frac{(2 \times 2)}{(3 \times 1)} = \frac{4}{3} = 1 \frac{1}{3}$  Note: dividing by half doubled the answer.

## Question 2

1. Find the reciprocal of  $2 \frac{2}{5}$

2.  $\frac{2}{3} \times \frac{7}{13} =$

3.  $1 \frac{1}{6} \times \frac{2}{9} =$

4.  $\frac{3}{7} \div \frac{2}{5} =$

5.  $2 \frac{2}{5} \div 3 \frac{8}{9} =$

6.  $\frac{(-25) \div (-5)}{4 - 2 \times 7} =$

7.  $\frac{-7}{2} \div \frac{-4}{9} =$

8. If we multiply 8 and the reciprocal of 2, what do we get?

9. Which is the better score in a geology test; 17 out of 20 or 22 out of 25?

10. Half-life is the time it takes for one-half of a radioactive element to decay. For example, the half-life of Cobalt-60 is 5.26 years.

- If you start with 8 g of Cobalt-60, how much will remain after 5.26 years (or one half-life)?
- How much will remain after 2 half-lives?

# 3. Converting Decimals & Fractions

## Converting Decimals into Fractions

Decimals are an almost universal method of displaying data, particularly given that it is easier to enter decimals, rather than fractions, into computers. But fractions can be more accurate. For example,  $\frac{1}{3}$  is not 0.33 it is  $0.3\dot{3}$


The method used to convert decimals into fractions is based on the notion of place value. The place value of the last digit in the decimal determines the denominator: tenths, hundredths, thousandths, and so on...

### Example problems:

- a) 0.5 has 5 in the tenths column. Therefore, 0.5 is  $\frac{5}{10} = \frac{1}{2}$  (simplified to an equivalent fraction).
- b) 0.375 has the 5 in the thousandth column. Therefore, 0.375 is  $\frac{375}{1000} = \frac{3}{8}$
- c) 1.25 has 5 in the hundredths column and you have  $1\frac{25}{100} = 1\frac{1}{4}$

The hardest part is converting to the lowest equivalent fraction. If you have a scientific calculator, you can use the fraction button. This button looks different on different calculators so read your manual if unsure.

If we take  $\frac{375}{1000}$  from example 2 above:

Enter 375 then  followed by 1000 press = and answer shows as  $\frac{3}{8}$ .

NOTE: The calculator does not work for rounded decimals; especially thirds. E.g,  $0.333 \approx \frac{1}{3}$

This table lists some commonly encountered fractions expressed in their decimal form:

Decimal	Fraction	Decimal	Fraction
0.125	$\frac{1}{8}$	0.5	$\frac{1}{2}$
0.25	$\frac{1}{4}$	0.66667	$\frac{2}{3}$
0.33333	$\frac{1}{3}$	.75	$\frac{3}{4}$
0.375	$\frac{3}{8}$	0.2	$\frac{1}{5}$

### Question 3:

Convert to fractions (no Calculator first, then check).

- a) 0.65 =
- b) 2.666 =
- c) 0.54 =
- d) 3.14 =
- e) What is 40 multiplied by 0.2? (use your knowledge of fractions to solve)



## Converting Fractions into Decimals

Converting fractions into decimals is based on place value. Using the concept of equivalent fractions, we can easily convert  $\frac{2}{5}$  into a decimal. First we convert to a denominator that has a 10 base:

$$\frac{2}{5} \text{ into tenths} \rightarrow \frac{2 \times 2}{5 \times 2} = \frac{4}{10} \therefore \text{we can say that two fifths is the same as four tenths: } 0.4$$

Converting a fraction to decimal form is a simple procedure because we simply use the divide key on the calculator.

Note: If you have a mixed number, convert it to an improper fraction before dividing it on your calculator.

### Example problems:

$$\triangleright \frac{2}{3} = 2 \div 3 = 0.6666666666 \dots \approx 0.67$$

$$\triangleright \frac{3}{8} = 3 \div 8 = 0.375$$

$$\triangleright \frac{17}{3} = 17 \div 3 = 5.6666666 \dots \approx 5.67$$

$$\triangleright 3\frac{5}{9} = (27 + 5) \div 9 = 3.555555556 \dots \approx 3.56$$

## Questions 4

Convert to decimals. Round your answer to three decimal places where appropriate.

a.  $\frac{17}{23} =$

b.  $\frac{5}{72} =$

c.  $56\frac{2}{3} =$

d.  $\frac{29}{5} =$



Watch this short Khan Academy video for further explanation:

**“Converting fractions to decimals”** (and vice versa)

<https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/decimal-to-fraction-pre-alg/v/converting-fractions-to-decimals>

## 4. Percentage

The concept of percentage is an extension of the material we have already covered about fractions. To allow comparisons between fractions we need to use the same denominator. As such, all percentages use 100 as the denominator. The word percent or “per cent” means per 100. Therefore, 27% is  $\frac{27}{100}$ .

To use percentage in a calculation, the simple mathematical procedure is modelled below:

$$\text{For example, } 25\% \text{ of } 40 \text{ is } \frac{25}{100} \times 40 = 10$$

Percentages are most commonly used to compare parts of an original. For instance, the phrase ‘30% off sale,’ indicates that whatever the original price, the new price is 30% less. However, the question might be more complex, such as, “How much is left?” or “How much was the original?”

### Example problems:

- a. An advertisement at the chicken shop states that on Tuesday everything is 22% off. If chicken breasts are normally \$9.99 per kilo. What is the new per kilo price?

Step 1: SIMPLE PERCENTAGE:

$$\frac{22}{100} \times 9.99 = 2.20$$

Step 2: DIFFERENCE: Since the price is cheaper by 22%, \$2.20 is subtracted from the original:  
 $9.99 - 2.20 = \$7.79$

- b. A new dress \$237, reduced from \$410. What is the percentage difference? As you can see, the problem is in reverse, so we approach it in reverse.

Step 1: DIFFERENCE: Since it is a discount the difference between the two is the discount. Thus we need to subtract \$237.00 from \$410 to see what the discount was that we received.

$$\$410 - \$237 = \$173$$

Step 2: SIMPLE PERCENTAGE: now we need to calculate what percentage of \$410 was \$173, and so we can use this equation:  $\frac{x}{100} \times 410 = 173$

We can rearrange the problem in steps:  $\frac{x}{100} \times 410 \div 410 = 173 \div 410$  this step involved dividing

410 from both sides to get  $\frac{x}{100} = \frac{173}{410}$  Next we work to get the  $x$  on its own, so we multiply both sides by 100.

Now we have  $x = \frac{173}{410} \times \frac{100}{1}$  Next we solve, so 0.42 multiplied by 100,  $\therefore 0.42 \times 100$  and we get

42.  $\therefore$  The percentage difference was 42%.

Let’s check: 42% of \$410 is \$173,  $\$410 - \$173 = \$237$ , the cost of the dress was \$237.00 ✓.

### Question 5

- a) GST adds 10% to the price of most things. How much does a can of soft drink cost if it is 80c before GST?

- b) Grains in a sandstone sample are counted and classified. Recalculate the grain counts to percentage.

Grain type	Count	Percentage
Quartz	254	
Feldspar	17	
Lithic fragment	29	

- c) Which of the following is the largest?  $\frac{3}{5}$  or  $\frac{16}{25}$  or 0.065 or 63%? (Convert to percentages)

# 5. Ratios

A ratio is a comparison of the size of one number to the size of another number. A ratio represents for every determined amount of one thing, how much there is of another thing. Ratios are useful because they are unit-less. That is, the relationship between two numbers remains the same regardless of the units in which they are measured.

Ratios use the symbol  $:$  to separate quantities being compared. For example, 1:3 means 1 unit to 3 units.



There is 1 red square to 3 blue squares

**1:3**

**1 to 3**

Ratios can be expressed as fractions but you can see from the above diagram that 1:3 is not the same as  $\frac{1}{3}$ . The fraction equivalent is  $\frac{1}{4}$

**Example:**

A pancake recipe requires flour and milk to be mixed to a ratio of 1:3. This means one part flour to 3 parts milk. No matter what device is used to measure, the ratio must stay the same.

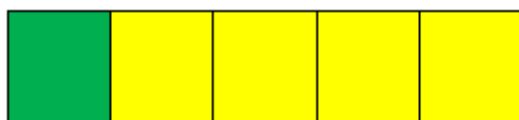
So if I add 200 mL of flour, I add  $200 \text{ mL} \times 3 = 600 \text{ mL}$  of milk

If I add 1 cup of flour, I add 3 cups of milk

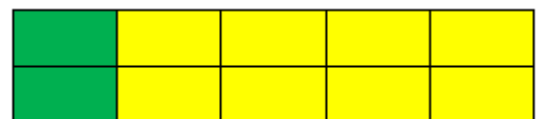
If I add 50 grams of flour, I add 150 grams of milk

*Scaling ratios*

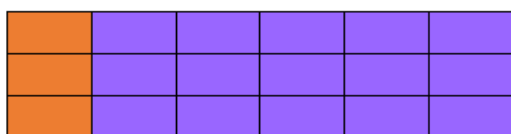
A ratio can be scaled up:



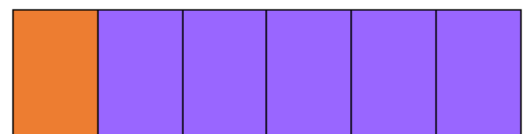
$1:4 = 2:8$



Or scaled down:



$3:15 = 1:5$



1:5 is the same as      2:10 is the same as      3:15 is the same as      4:20 and so on

Scaling ratios is useful in the same way that simplifying fractions can be helpful, for example, in comparing values. For ratios the same process as simplifying fractions is applied – that is, scaling must be applied to both numbers.

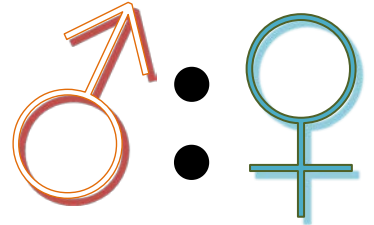
### Example

A first year physiology subject has 36 males and 48 females, whereas the endocrinology subject has 64 males and 80 females. You are asked to work out which cohort has the largest male to female ratio.

The male: female ratios can be expressed as:

36:48 – Earth science subject

64:80 – chemistry subject



Both numbers of the ratio 36:48 can be divided by 12 to leave the ratio 3:4

Both numbers of the ratio 64:80 can be divided by 16 to leave the ratio 4:5

These two ratios cannot be easily directly compared, but they can be rescaled to a common value of 20 for the females, against which the males can be compared if they are rescaled equivalently.

$3 (x5):4 (x5) = 15:20$  – Earth science subject

$4 (x4):5 (x4) = 16:20$  – chemistry subject

Comparing the ratios now shows that the chemistry subject has a slightly higher ratio of males to females.

### Questions 6:

- For a 1:5 concentration of cordial drink, how much cordial concentrate do I have to add to water to make up a 600 mL jug?
- An orange paint is made up of a mixture of red and yellow paint in the ratio 3:5. If you have 15 cups of yellow paint, how many cups of red paint do you need to make the correct hue of orange?
- Which of the following ratios is the odd one out? 9:27, 3:9, 8:28, 25:75

## Ratio Scales

Ratios are a useful way to define scale. When we want to represent something at a scale smaller than it is in reality, we can use a ratio scale to quantify how much the image or model is scaled. For example, a map shows geographic features at a scale much smaller than reality.

The ratio scale for a map may be expressed as 1: 10 000, which indicates something measured as 1 unit on the map is x 10 000 bigger in the real world.

Remember, ratios are dimensionless (i.e. don't have units). When applying ratios, use the same unit for both sides.

### EXAMPLE PROBLEM

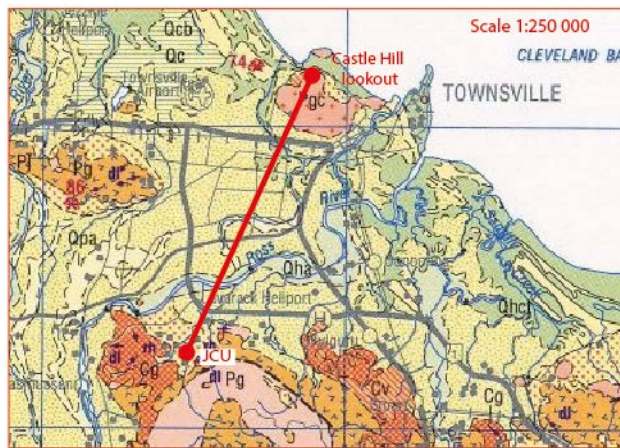
For a map with a ratio scale of 1:250 000, 1 cm on the map is equivalent to 250 000 cm in real life. The distance between **JCU** and **Castle Hill lookout** on the map shown is 4 cm (check with your ruler).

Using the ratio scale 1:250 000, we can scale up by multiplying both sides by 4:

$$1 \text{ (x4)}:250\,000\text{(x4)}$$

Giving  $\rightarrow$  4:1 000 000

In other words: 4 cm on the map represents 1 000 000 cm in real life. Since we don't usually use centimetres as a unit of measure for large numbers, we can convert 1 000 000 cm to something more familiar.



To convert cm to km we know:

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ km} = 1000 \text{ m}$$

$$\text{So } 1\,000\,000 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 10 \text{ km}$$

So the straight-line distance in real life between JCU and Castle Hill lookout is 10 km.

### Questions 7:

- A 1:20 scale model boat is 15 cm wide. How wide is the actual boat?
- If the boat mast is 4 m high, how high is the mast on the model?
- The scale of a map is 1:10 000. The straight-line distance on a map from point A to point B is 3 cm. What is the real-world distance between these points?
- The scale of a map is 1:50 000. What length on the map will represent 5 km?

### Whole ratios and proportion

Ratios can be expressed as a fraction when one part of a population is compared against the whole population. This is called a *whole ratio*.

If a population is divided into children and adults, we can express the relationship as children:adults.

Another way to show the relationship between children and adults is to show the proportion of the children or adults to the whole population.

$$\frac{\text{children}}{\text{children+adults}} = \frac{\text{children}}{\text{whole population}} \quad \text{or} \quad \frac{\text{adults}}{\text{children+adults}} = \frac{\text{adults}}{\text{whole population}}$$

Showing ratios in this way is useful if we want to calculate an unknown value from an equivalent population that has the same proportional ratio.

#### Example:

The average ratio of children to adults in refugee camps is 3:1. If an influx of 1000 new refugees are anticipated in a camp, how many child immunisation doses may be required?

The known ratio is:  $\frac{3 \text{ children}}{3 \text{ children} + 1 \text{ adult}} = \frac{3}{4}$

So the anticipated (unknown) number of children can be calculated as an equivalent ratio.

$\frac{3}{4} = \frac{\text{unknown}}{1000}$  4 is multiplied by 250 to make 1000, so we multiply 3 by the same amount

$$\frac{3 (\times 250)}{4 (\times 250)} = \frac{750}{1000}$$

750 children should be anticipated in the new influx, so immunisation doses can be ordered accordingly.

#### Questions 8:

- a) Jane reads 25 pages in 30 minutes. How long does it take her to read 200 pages?
  
  
  
  
  
  
  
  
  
  
- b) The shadow from a tree is 12 metres long. Nearby, a 30 cm ruler held vertically casts a shadow of 40 cm. How tall is the tree?
  
  
  
  
  
  
  
  
  
  
- c) It takes 1500 years for 1 metre of sediment to accumulate in a lake. How many years does it take 3 metres of sediment to accumulate? What is the age of the sediment at 3 metres depth?

## 6. Algebra Refresh

### Addition and Multiplication Properties

Maths Property	Rule	Example
<b>Commutative</b> The number order for addition or multiplication doesn't affect the sum or product	$a + b = b + a$	$1 + 3 = 3 + 1$
	$ab = ba$	$2 \times 4 = 4 \times 2$
<b>Associative</b> Since the Number order doesn't matter, it may be possible to regroup numbers to simplify the calculation	$a + (b + c) = (a + b) + c$	$1 + (2 + 3) = (1 + 2) + 3$
	$a(bc) = (ab)c$	$2 \times (2 \times 3) = (2 \times 2) \times 3$
<b>Distributive</b> A factor outside the bracket can be multiplied with individual terms within a bracket to give the same result	$a(b + c) = ab + ac$	$2(3 + 1) = 2 \times 3 + 2 \times 1$
<b>Zero Factor</b>	$a \times 0 = 0$ If $ab = 0$ , then either $a = 0$ or $b = 0$	$2 \times 0 = 0$
<b>Rules for Negatives</b>	$-(-a) = a$ $(-a)(-b) = ab$ $-ab = (-a)b = a(-b)$ $= -(ab)$ $(-1)a = -a$	$-(-3) = 3$ $(-2)(-3) = 2 \times 3$ $-2 \times 3 = (-2) \times 3$ $= 2 \times (-3)$ $= -(2 \times 3)$ $(-1) \times 2 = -2$
<b>Rules for Division</b>	$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$  $\frac{-a}{-b} = \frac{a}{b}$	$-\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2}$  $\frac{-6}{-3} = \frac{6}{3}$

Additional help with algebra is available from the [Algebra Basics](#) module, downloadable from [The Learning Centre](#).



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If  $\frac{a}{b} = \frac{c}{d}$  then  $ad = bc$

Proof:

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{b \times a}{b} = \frac{bc}{d} \text{ (multiply everything by } b)$$

$$a = \frac{bc}{d}$$

$$a \times d = \frac{bc \times d}{d} \text{ (multiply by } d)$$

$$ad = bc$$

If  $\frac{1}{2} = \frac{3}{4}$  then  $1 \times 4$

$$= 2 \times 3$$

### Question 9:

a) Simply the following

i.  $3(x-1)=2(x+5)$

ii.  $2x + 8 = \frac{1}{2}(43 + x)$

iii.  $2x^2 - 3x^3 - x^2 + 2x =$

iv.  $\frac{2(4+6)}{2+3} =$

b) Expand  $2y^2(3x + 7y + 2)$

c) What is the molecular mass of  $\text{H}_2\text{O}_2$  Given  $O = 16$  and  $H = 1$

d) If the volume ( $V$ ) of the Earth is  $1.08 \times 10^{21} \text{ m}^3$ , what is the radius ( $r$ ), given:

$$V = \frac{4}{3}\pi r^3$$

**If you are not confident with using scientific notation, work through Section 8 first and then come back to this question.**



# 7. Power Operations

**Powers** are also called **exponents** or **indices**; we can work with **indices** to simplify expressions and to solve problems.

*Some key ideas:*

- a) Any base number raised to the power of 1 is the base itself: for example,  $5^1 = 5$
- b) Any base number raised to the power of 0 equals 1, so:  $4^0 = 1$
- c) Powers can be simplified if they are **multiplied** or **divided** and have the **same** base.
- d) Powers of powers are multiplied. Hence,  $(2^3)^2 = 2^3 \times 2^3 = 2^6$
- e) A negative power indicates a reciprocal:  $3^{-2} = \frac{1}{3^2}$

Certain rules apply and are often referred to as: **Index Laws**.

*Below is a summary of the index rules:*

Index Law	Substitute variables for values
$a^m \times a^n = a^{m+n}$	$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
$a^m \div a^n = a^{m-n}$	$3^6 \div 3^3 = 3^{6-3} = 3^3 = 27$
$(a^m)^n = a^{mn}$	$(4^2)^5 = 4^{2 \times 5} = 4^{10} = 1048576$
$(ab)^m = a^m b^m$	$(2 \times 5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100$
$(a/b)^m = a^m \div b^m$	$(10 \div 5)^3 = 2^3 = 8$ ; $(10^3 \div 5^3) = 1000 \div 125 = 8$
$a^{-m} = \frac{1}{a^m}$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$\frac{1}{a^m} = \frac{1}{a^m}$	$8^{1/3} = \sqrt[3]{8} = 2$
$a^0 = 1$	$6^3 \div 6^3 = 6^{3-3} = 6^0 = 1$ ; $(6 \div 6 = 1)$

**EXAMPLE PROBLEMS:**

a) Simplify  $6^5 \times 6^3 \div 6^2 \times 7^2 + 6^4 =$   
 $= 6^{5+3-2} \times 7^2 + 6^4$   
 $= 6^6 \times 7^2 + 6^4$

b) Simplify  $g^5 \times h^4 \times g^{-1} =$   
 $= g^5 \times g^{-1} \times h^4$   
 $= g^4 \times h^4$



Watch this short Khan Academy video for further explanation:  
**"Simplifying expressions with exponents"**  
<https://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/simplifying-expressions-with-exponents>

### Question 10:

a. Apply the index laws/rules:

i. Simplify  $5^2 \times 5^4 + 5^2 =$

ii. Simplify  $x^2 \times x^5 =$

iii. Simplify  $4^2 \times t^3 \div 4^2 =$

iv. Simplify  $(5^4)^3 =$

v. Simplify  $\frac{2^4 3^6}{3^4} =$

vi. Simplify  $3^2 \times 3^{-5} =$

vii. Simplify  $\frac{9(x^2)^3}{3xy^2} =$

viii. Simplify  $a^{-1}\sqrt{a} =$

b. What is the value of  $x$  for the following?

i.  $49 = 7^x$

ii.  $\frac{1}{4} = 2^x$

iii.  $88 = 11^1 \times 2^x$

iv.  $480 = 2^x \times 3^1 \times 5^1$

v. Show that  $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$

## 8. Scientific Notation

Numbers as multiples or fractions of ten	Number	Number as a power of ten
$10 \times 10 \times 10$	1000	$10^3$
$10 \times 10$	100	$10^2$
10	10	$10^1$
$10 \times 1/10$	1	$10^0$
$1/10$	0.1	$10^{-1}$
$1/100$	0.01	$10^{-2}$
$1/1000$	0.001	$10^{-3}$

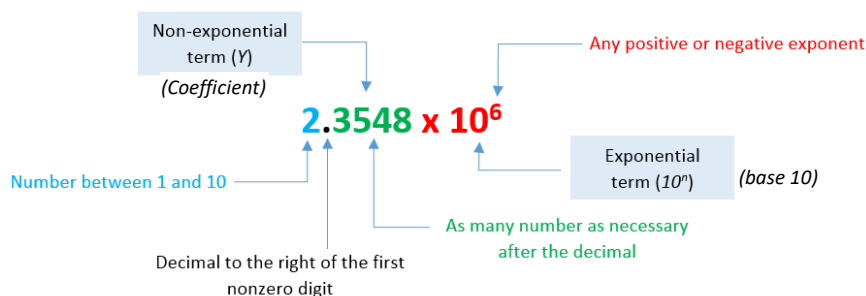
Scientific notation is a convenient method of representing and working with very large and very small numbers. Transcribing a number such as 0.000000000000082 or 5480000000000 can be frustrating since there will be a constant need to count the number of zeroes each time the number is used. Scientific notation provides a way of writing such numbers easily and accurately.

Scientific notation requires that a number is presented as a non-zero digit followed by a decimal point and then a power (exponential) of base 10. The exponential is determined by counting the number places the decimal point is moved.

The number 65400000000 in scientific notation becomes  $6.54 \times 10^{10}$ .

The number 0.00000086 in scientific notation becomes  $8.6 \times 10^{-7}$ .

(Note:  $10^{-6} = \frac{1}{10^6}$ .)



If  $n$  is **positive**, shift the decimal point that many places to the **right**.

If  $n$  is **negative**, shift the decimal point that many places to the **left**.

**Question 11:**

**Write the following in scientific notation:**

a. 450

b. 90000000

c. 3.5

d. 0.0975

**Write the following numbers out in full:**

e.  $3.75 \times 10^2$

f.  $3.97 \times 10^1$

g.  $1.875 \times 10^{-1}$

h.  $-8.75 \times 10^{-3}$

*Multiplication* and *division* calculations of quantities expressed in scientific notation follow the index laws since they all have the common base, i.e. base 10.

Here are the steps:

Multiplication	Division
A. <b>Multiply</b> the coefficients	1. <b>Divide</b> the coefficients
B. <b>Add</b> their exponents	2. <b>Subtract</b> their exponents
C. Convert the answer to scientific Notation	3. Convert the answer to scientific Notation
<b>Example:</b> $(7.1 \times 10^{-4}) \times (8.5 \times 10^{-5})$ $7.1 \times 8.5 = 60.35$ (multiply coefficients) $10^{-4} \times 10^{-5} = 10^{(-4+ -5)=-9}$ (add exponents) $= 60.35 \times 10^{-9}$ – check it's in scientific notation ✗ $= 6.035 \times 10^{-8}$ – convert to scientific notation ✓	<b>Example:</b> $(9 \times 10^{20}) \div (3 \times 10^{11})$ $9 \div 3 = 3$ (divide coefficients) $10^{20} \div 10^{11} = 10^{(20-11)=9}$ (subtract exponents) $= 3 \times 10^9$ – check it's in scientific notation ✓

Recall that addition and subtraction of numbers with exponents (or indices) requires that the base and the exponent are the same. Since all numbers in scientific notation have the same base 10, for *addition* and *subtraction* calculations, we have to adjust the terms so the exponents are the same for both. This will ensure that the digits in the coefficients have the correct place value so they can simply be added or subtracted.

Here are the steps:

Addition	Subtraction
1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent	1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent
2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places	2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places
3. Add the new coefficients	3. Subtract the new coefficients
4. Convert the answer to scientific notation	4. Convert the answer to scientific notation
<b>Example:</b> $(3 \times 10^2) + (2 \times 10^4)$ $4 - 2 = 2$ increase the small exponent by 2 to equal the larger exponent 4 $0.03 \times 10^4$ the coefficient of the first term is adjusted so its exponent matches that of the second term $= (0.03 \times 10^4) + (2 \times 10^4)$ the two terms now have the same base and exponent and the coefficients can be added $= 2.03 \times 10^4$ check it's in scientific notation ✓	<b>Example:</b> $(5.3 \times 10^{12}) - (4.224 \times 10^{15})$ $15 - 12 = 3$ increase the small exponent by 3 to equal the larger exponent 15 $0.0053 \times 10^{15}$ the coefficient of the first term is adjusted so its exponent matches that of the second term $= (0.0053 \times 10^{15}) - (4.224 \times 10^{15})$ the two terms now have the same base and exponent and the coefficients can be subtracted. $= -4.2187 \times 10^{15}$ check it's in scientific notation ✓

### Questions 12:

- a)  $(4.5 \times 10^{-3}) \div (3 \times 10^2)$
- b)  $(2.25 \times 10^6) \times (1.5 \times 10^3)$
- c)  $(6.078 \times 10^{11}) - (8.220 \times 10^{14})$  (give answer to 4 significant figures).
- d)  $(3.67 \times 10^5) \times (23.6 \times 10^4)$
- e)  $(7.6 \times 10^{-3}) + (\sqrt{9.0 \times 10^{-2}})$
- f) Two particles weigh  $2.43 \times 10^{-2}$  grams and  $3.04 \times 10^{-3}$  grams. What is the difference in their weight in scientific notation?
- g) How long does it take light to travel to the Earth from the Sun in seconds, given that the Earth is  $1.5 \times 10^8$  km from the Sun and the speed of light is  $3 \times 10^5$  km/s?
- h) The first great oxygenation of Earth's atmosphere occurred about 2.6 billion years ago (Ga) and the onset of complex life began at the beginning of the Cambrian, 542 million years ago (Ma). How many millions of years passed between oxygen rising in the atmosphere and the evolution of respiring ecosystems represented by complex life?

## 9. Units and Unit Conversion

Measurement is used every day to describe quantity. There are various types of measurements such as time, distance, speed, weight and so on. There are also various systems of units of measure, for example, the Metric system and the Imperial system. Within each system, for each base unit, other units are defined to reflect divisions or multiples of the base unit. This is helpful for us to have a range of unit terms that reflect different scale

Measurements consist of two parts – the number and the identifying unit.



In scientific measurements, units derived from the metric system are the preferred units. The metric system is a decimal system in which larger and smaller units are related by factors of 10.

Common Prefixes of the Metric System

Prefix	Abbreviation	Relationship to Unit	Exponential Relationship to Unit	Example
mega-	M	1 000 000 x Unit	$10^6$ x Unit	2.4ML -Olympic sized swimming pool
kilo-	k	1000 x Unit	$10^3$ x Unit	The average newborn baby weighs 3.5kg
-	-	Units	Unit	metre, gram, litre, sec
deci-	d	1/10 x Unit or 0.1 x Unit	$10^{-1}$ x Unit	2dm - roughly the length of a pencil
centi-	c	1/100 x Unit or 0.01 x Unit	$10^{-2}$ x Unit	A fingernail is about 1cm wide
milli-	m	1/1000 x Unit or 0.001 x Unit	$10^{-3}$ x Unit	A paperclip is about 1mm thick
micro-	$\mu$	1/1 000 000 x Unit or 0.000001 x Unit	$10^{-6}$ x Unit	human hair can be up to 181 $\mu$ m
nano-	n	1/1 000 000 000 x Unit or 0.000000001 x Unit	$10^{-9}$ x Unit	DNA is 5nm wide

## Common Metric Conversions

Unit	Larger Unit	Smaller Unit
1 metre	1 kilometre = 1000 metres	100 centimetres = 1 metre 1000 millimetres = 1 metre
1 gram	1 kilogram = 1000 grams	1000 milligrams = 1 gram 1 000 000 micrograms = 1 gram
1 litre	1 kilolitre = 1000 litres	1000 millilitres = 1 litre

Converting units of measure may mean converting between different measuring systems, such as from imperial to metric (e.g. mile to kilometres), or converting units of different scale (e.g. millimetres to metres). In the fields of science, converting measurement can be a daily activity.

It helps to apply a formula to convert measurements. However, it is essential to understand the how and why of the formula otherwise the activity becomes one simply committed to memory without understanding what is really happening. Most mistakes are made when procedures are carried out without understanding the context, scale or purpose of the conversion.

### *Unit Conversion rules:*

- I. Always write the unit of measure associated with every number.*
- II. Always include the units of measure in the calculations.*

Unit conversion requires algebraic thinking.

E.g. Converting 58mm into metres:  $58 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 0.058 \text{ m}$

The quantity  $\frac{1 \text{ m}}{1000 \text{ mm}}$  is called a **conversion factor**; it is a division/quotient; in this case it has metres on top and mm on the bottom. The conversion factor simply functions to create an equivalent fraction, as 1m is equal to 1000 mm and any value divided by itself is equal to 1.

We work with information given, a conversion factor, and then the desired unit.





### Example problem:

Convert 125 milligrams (mg) to grams (g). There are 1000 mg in a gram so our conversion factor is

$$\frac{1 g}{1000 mg}$$

The working is as follows:  $125 \text{ mg} \times \frac{1 g}{1000 \text{ mg}} = 0.125 g$

Here the mg cancels out leaving g, which is the unit we were asked to convert to.

It is helpful to have a thinking process to follow. This one comes from the book, *Introductory Chemistry* (Tro, 2011, pp. 25-35). There are four steps to the process: sort, strategise, solve and check.

1. **Sort:** First we sort out what given information is available.
2. **Strategise:** The second step is where a conversion factor is created. You may need to look at a conversion table and possibly use a combination of conversion factor.
3. **Solve:** This is the third step which simply means to solve the problem.
4. **Check:** The last step requires some logical thought; does the answer make sense?
- 5.

**Example problem:** Convert 2 kilometres (km) into centimetres (cm).

- **Sort:** we know there are 1000 metres in one km, and 100cm in one metre.
- **Strategise:** So our conversion factors could be  $\frac{1000m}{1km}$  and  $\frac{100cm}{1m}$
- **Solve:**  $2 km \times \frac{1000 m}{1km} \times \frac{100 cm}{1m} = x cm$   
 $2 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{ m}} = 2 \times 1000 \times 100 \text{ cm} \therefore 2 \text{ km} = 200,000 \text{ cm}$
- **Check:** is there 200,000cm in a kilometre? Yes, that seems sensible.

### **Question 13:**

Convert the following:

- a) 0.25 g to milligrams
- b) 15  $\mu\text{g}$  to mg
- c) 67 mL to L
- d) Using this information:  $1 m^2 = 10000cm^2 = 1000000mm^2$   
Convert  $1.5m^2$  to  $mm^2$

$$1m = 100cm = 1000mm$$

**More Examples:**

a) Convert 0.15 g to kilograms and milligrams	b) Convert 5234 mL to litres
<p>Because 1 kg = 1000 g, 0.15 g can be converted to kilograms as shown:</p> $0.15 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 0.00015 \text{ kg}$ <p>Also, because 1 g = 1000 mg, 0.15 g can be converted to milligrams as shown:</p> $0.15 \text{ g} \times \frac{1000 \text{ mg}}{1 \text{ g}} = 150 \text{ mg}$	<p>Because 1 L = 1000 mL, 5234 mL can be converted to litres as shown:</p> $5234 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}} = 5.234 \text{ L}$

**Question 14:**

- a) Convert 120 g to kilograms and milligrams. Use scientific notation for your answer.
- b) Convert 4.264 L to kilolitres and millilitres
- c) Convert 670 micrograms to grams. Give your answer in scientific notation.
- d) How many millilitres are in a cubic metre?
- e) How many inches in 38.10cm (2.54cm = 1 inch)
- f) How many centimetres in 1.14 kilometres?
- g) How millions of years are 1.6 billion years?



Watch this short Khan Academy video for further explanation:

**“Unit conversion word problem: drug dosage”**

<https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-unit-conversion/v/unit-conversion-example-drug-dosage>

# 10. Logarithms

$$4^2$$

- i. The 2 is called the **exponent, index or power**.
- ii. The 4 is called the **base**.

With roots we tried to find the unknown base. Such as,  $x^3 = 64$  is the same as  $\sqrt[3]{64} = x$ ; ( $x$  is the base).

A **logarithm** is used to find an unknown **power/exponent**. For example,  $4^x = 64$  is the same as  $\log_4 64 = x$

This example above is spoken as: 'The logarithm of 64 with base 4 is  $x$ .' The base is written in subscript.

The general rule is:  $N = b^x \Leftrightarrow \log_b N = x$

- i. In mathematics the base can be any number, but only two types are commonly used:
  - a.  $\log_{10} N$  (*base 10*) is often abbreviated as simply Log, and
  - b.  $\log_e N$  (*base e*) is often abbreviated as Ln or natural log
- ii.  $\log_{10} N$  is easy to understand for:  $\log_{10} 1000 = \log 1000 = 3$  ( $10^3 = 1000$ )  
 $\log 100 = 2$
- iii. Numbers which are not 10, 100, 1000 and so on are not so easy. For instance,  $\log 500 = 2.7$  It is more efficient to use the calculator for these types of expressions.

**Question 15** Write the logarithmic form for:

- a)  $5^2 = 25$
- b)  $6^2 = 36$
- c)  $3^5 = 243$

Use your calculator to solve

- d)  $\text{Log } 10000 =$
- e)  $\text{Log } 350 =$
- f)  $\text{Ln } 56 =$
- g)  $\text{Ln } 100 =$

Use the provided log laws to simplify the following:

- i)  $\log 5 + \log 6$
- j)  $\log 4 - \log 8$
- k)  $2\log 4 - 2\log 2$



$\text{Log}_a(x) = y$  then  $a^x = y$

$\log_b(xy) = \log_b(x) + \log_b(y)$

$\log_b(x/y) = \log_b(x) - \log_b(y)$

$\log_b(x^y) = y \cdot \log_b(x)$

$a \cdot \log_b(x) = \log_b(x^a)$

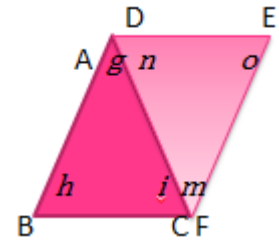
Some log rules for your reference

# 11. Trigonometry

Trigonometry is the study of the properties of triangles, as the word suggests. Considering that all polygons can be divided into triangles, understanding properties of triangles is important. Trigonometry has applications for a range of science and engineering subjects.

## Congruence

If two plane shapes can be placed on top of each other exactly, then they are **congruent**. The corresponding **angles**, for example,  $\angle BAC$  and  $\angle DFE$  ( $g$  and  $m$ ) are the same, and the **intervals**, for instance,  $AB$  and  $FE$  of each shape match perfectly. In addition, the perimeter and the area will be identical, thus the perimeter and area of  $\triangle ABC$  is identical to the perimeter and area of  $\triangle DEF$ . Also, the vertices and sides will match exactly for  $\triangle ABC$  &  $\triangle DEF$ .



In summary:  $g = m$ ;  $h = o$ ;  $i = n$   
 $AB = EF$ ;  $BC = ED$ ;  $AC = FD$

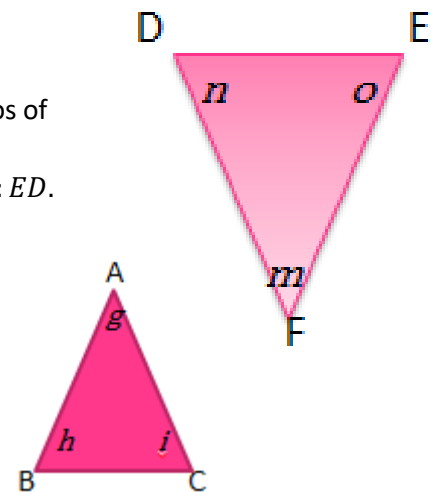
The mathematical symbol for congruence is " $\cong$ ". Therefore,  $\triangle ABC \cong \triangle DEF$

Many geometric principles involve ideas about congruent triangles.

## Similar

If a shape looks that same but is a different size it is said to be **similar**. The corresponding angles in shapes that are similar will be **congruent**. The ratios of adjacent sides in the corresponding angle will be the same for the ratio of adjacent sides in a similar triangle. For example: the ratio  $AB:BC$  is as  $EF:ED$ .

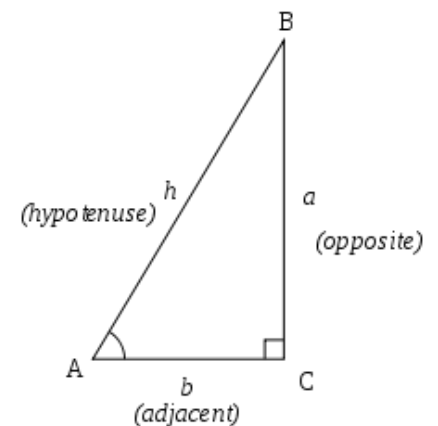
The angles are the same:  $g = m$ ;  $h = o$ ;  $i = n$



## Right Angle Triangles and the Pythagorean Theorem

To begin, we can identify the parts of a right angled triangle:

- The right angle is symbolised by a **square**.
- The side directly opposite the right angle is called the **hypotenuse**. The hypotenuse **only** exists for right angle triangles.
- The hypotenuse is always the longest side.
- If we focus on the angle  $\angle BAC$ , the side opposite is called the **opposite** side.
- The side touching angle  $\angle BAC$  is called the **adjacent** side.

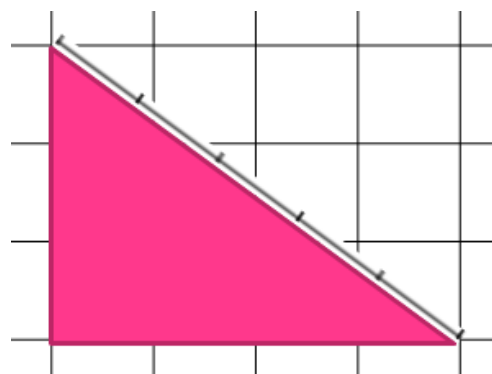


## Investigating right angle triangles

The **hypotenuse** is related to a Greek word that means to stretch. What do you do if you were to make a right angle and you did not have many measuring instruments? One way is to take a rope, mark off three units, turn; then four units, turn; and then five units. Join at the beginning and when stretched out, you should have a triangle. The triangle created will form a right angle, as right.

The relationship between these numbers, 3, 4 and 5 were investigated further by Greek mathematicians and are now commonly known now as a Pythagorean triple. Another triple is 5, 12 and 13. You might recall the mathematical formula:  $a^2 + b^2 = c^2$ . This is one of the rules of trigonometry; the Pythagoras theorem which states:

**“The square of the hypotenuse is equal to the sum of the square of the other two sides”**



### EXAMPLE PROBLEMS:

1. Calculate the length of the unknown side.

Step 1: Recognise the hypotenuse is the unknown  $c$ .

Step 2: Write the formula:  $c^2 = a^2 + b^2$

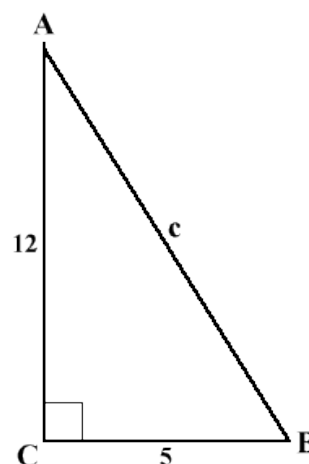
Step 3: Sub in the numbers  $c^2 = 5^2 + 12^2$

$$c^2 = 25 + 144$$

$$c^2 = 169$$

$$c = \sqrt{169} \text{ (The opposite of square is root)}$$

$$c = 13$$



2. Calculate the length of the unknown side.

Step1: Recognise the adjacent side is unknown  $a$ .

Step 2: Write the formula:  $c^2 = a^2 + b^2$

Step 3: Sub in the numbers  $20^2 = a^2 + 16^2$

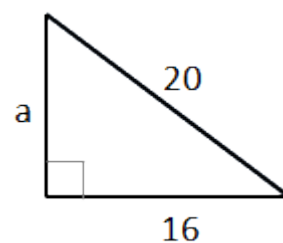
$$400 = a^2 + 256$$

$$400 - 256 = a^2 \text{ (Rearrange)}$$

$$144 = a^2$$

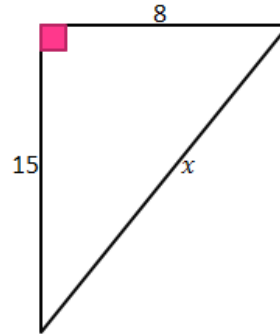
$$\sqrt{144} = a$$

$$12 = a$$

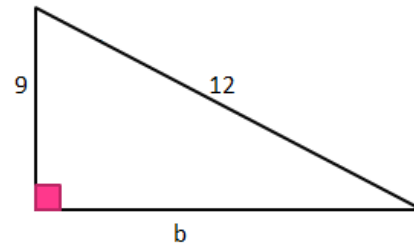


**Question 16:**

- a. Calculate the unknown side  $x$  :

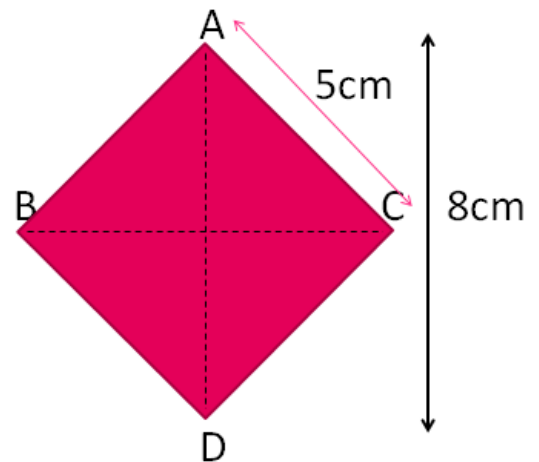


- b. Calculate the unknown side  $b$  correct to two places:



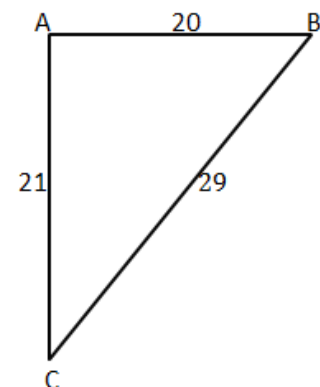
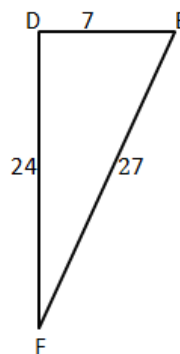
decimal

- c. A support wire is required to strengthen the sturdiness of a pole. The pole stands 20m and the wire will be attached to a point 15m away. How long will the wire be?
- d. Here we have a rhombus  $ABCD$ . It has a diagonal of 8cm. It has one side of 5cm.
- Find the length of the other diagonal (use Pythagoras Theorem)
  - Find the area of the rhombus.



- e. Which of the following triangles is a right angle triangle? Explain why.

- f. Name the right angle.



## Trigonometric Functions: Sine, Cosine and Tangent

In this section we extend on the Pythagoras Theorem, which relates to the three sides of a right angled triangle, to trigonometrical ratios, which help to calculate an angle of a triangle involving lengths and angles of right angle triangles. This is a basic introduction to trigonometry that will help you to explore the concept further in your studies.

Often angles are marked with ' $\theta$ ' which is the Greek letter 'theta'. This symbol helps to identify which angle we are dealing with.

For instance, to describe the right angle triangle (right)

The side BC is opposite  $\theta$

The side AC is adjacent  $\theta$

The opposite side of the right angle is the hypotenuse AB

The trigonometrical ratios do not have units of measure themselves – they are ratios. The three ratios are cosine of  $\theta$ , the sine of  $\theta$  and the tangent of  $\theta$ . These are most often abbreviated to sin, cos and tan.

We can calculate an angle when given one of its trigonometrical ratios.

The ratios depend on which angles and sides are utilised

The three ratios are:

$$\begin{aligned} \circ \sin A &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \circ \cos A &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ \circ \tan A &= \frac{\text{opposite}}{\text{adjacent}} \end{aligned}$$

If you are required to work with these ratios, you might like to memorise the ratios as an acronym SOHCAHTOA pronounced “sock – a- toe – a” or the mnemonic: “Some Old Humans Can Always Hide Their Old Age.”

**The formulas can be used to either find an unknown side or an unknown angle.**

### EXAMPLE PROBLEMS:

1. Find  $x$

Step 1: Label the triangle: 19m is the hypotenuse and  $x$  is adjacent to the  $60^\circ$

Step 2: Write the correct formula:  $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

Step 3: Sub in the numbers:  $\cos 60 = \frac{x}{19}$

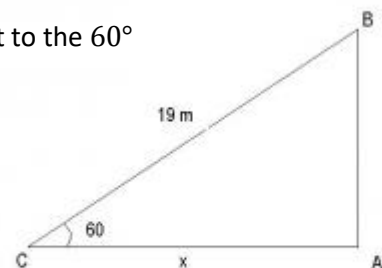
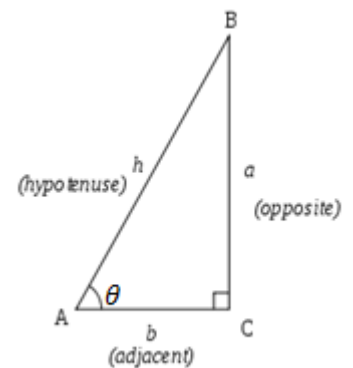
Step 4: Rearrange the formula:  $19 \cos 60 = x$

Step 5: Enter into calculator:  $9.5 = x$

Therefore, Side  $x = 9.5$  m

2. Find  $a$

Step 1: Label the triangle: 6.37in is opposite and 15.3in is adjacent to the angle



Step 2: Write the correct formula:  $\tan A = \frac{\textit{opposite}}{\textit{adjacent}}$

Step 3: Sub in the numbers:  $\tan a = \frac{6.37}{15.3}$

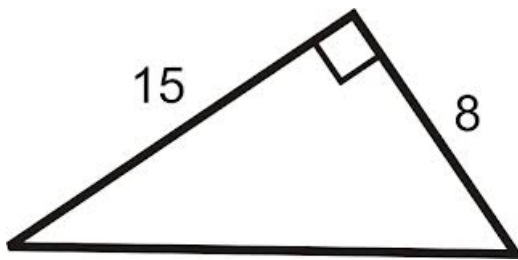
Step 4: Rearrange the formula:  $a = \tan^{-1}\left(\frac{6.37}{15.3}\right)$

Step 5: Enter into calculator:  $a = 22.6^\circ$

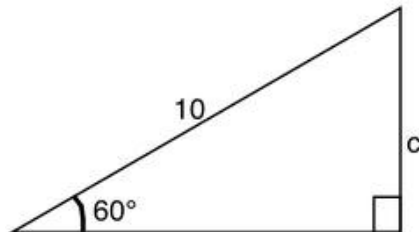


**Question 17:**

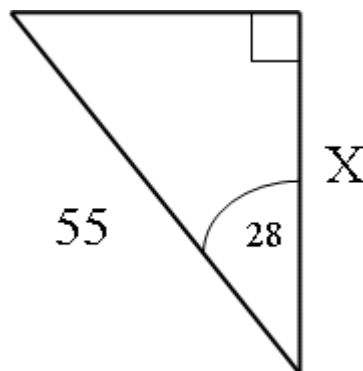
a) Find both angles in the triangle below.



b) Find C in the triangle below.



c) Find X in the triangle below.





## Applying Trigonometric Functions

Trigonometric Functions are used in a wide range of professions to solve measurement problems, e.g. architecture, cartography, navigation, land-surveying and engineering. Less obvious uses include the study of distances between stars in astronomy and more abstract applications in geophysics, medical imaging, seismology and optics. The sine and cosine functions are particularly important to the theory of periodic functions such as those that describe sound and light waves.

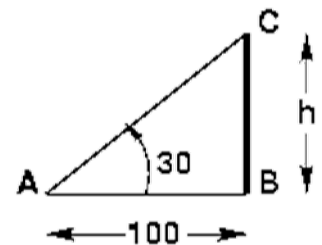
To simulate the real world application of trigonometric functions, you may be asked to solve word problems like the ones below in your exams.

### EXAMPLE PROBLEMS:

- 1) If the distance of a person from a tower is 100 m and the angle subtended by the top of the tower with the ground is  $30^\circ$ , what is the height of the tower in metres?

Step 1:

Draw a simple **diagram** to represent the problem. Label it carefully and clearly mark out the quantities that are given and those which have to be calculated. Denote the unknown dimension by say  $h$  if you are calculating height or by  $x$  if you are calculating distance.



Step 2:

Identify which trigonometric function represents a ratio of the side about which information is given and the side whose dimensions we have to find out. Set up a **trigonometric equation**.

AB = distance of the man from the tower = 100 m

BC = height of the tower =  $h$  (to be calculated)

The trigonometric function that uses AB and BC is  $\tan a$ , where  $a = 30^\circ$

Step 3:

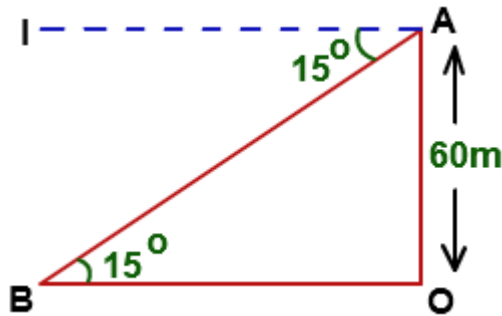
Substitute the value of the trigonometric function and **solve the equation** for the unknown variable.

$$\tan 30^\circ = \frac{BC}{AB} = \frac{h}{100m}$$

$$h = 100m \times \tan 30^\circ = 57.74m$$

- 2) From the top of a light house 60 metres high with its base at the sea level, the angle of depression of a boat is 15 degrees. What is the distance of the boat from the foot of the light house?

Step 1: **Diagram**



OA is the height of the light house

B is the position of the boat

OB is the distance of the boat from the foot of the light house

**Step 2: Trigonometric Equation**

$$\tan 15^\circ = \frac{OA}{OB}$$

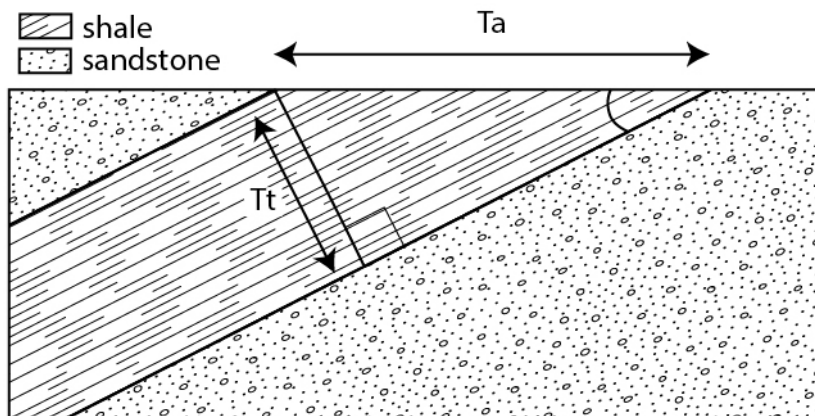
**Step 3: Solve the equation**

$$\tan 15^\circ = \frac{60m}{OB}$$

$$OB = 60m \div \tan 15^\circ = 223.92m$$

**Question 18**

- a) If your distance from the foot of the tower is 200m and the angle of elevation is  $40^\circ$ , find the height of the tower.
- b) A ship is 130m away from the centre of a barrier that measures 180m from end to end. What is the minimum angle that the boat must be turned to avoid hitting the barrier?
- c) Sedimentary beds are dipping  $27^\circ$  from horizontal. The thickness of the shale unit is measured horizontally at the surface with at the tape giving an apparent thickness ( $T_a$  on figure) of 30 m. What is the true thickness of the shale unit ( $T_t$  on the figure)?

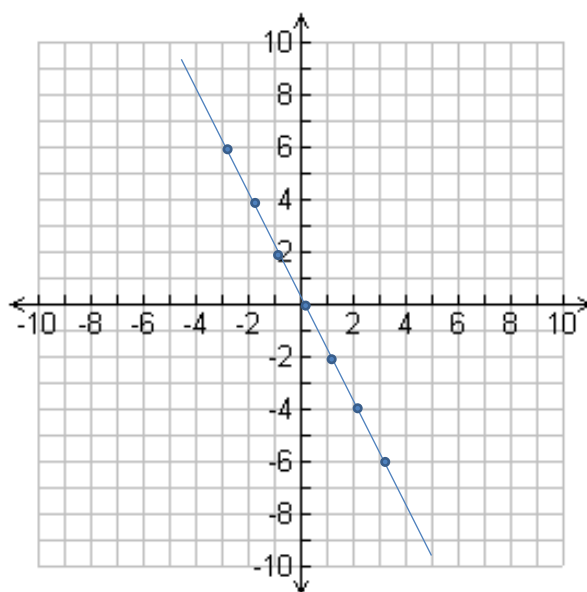


# 12. Graphs

## Linear Patterns

Cartesian Planes are often used to plot data points to see if there is a pattern in the data. A common method is using a table like the one below; the coordinates are plotted on the graph.

x value	-3	-2	-1	0	1	2	3
y value	6	4	2	0	-2	-4	-6
Coordinate	(-3, 6)	(-2, 4)	(-1, 2)	(0, 0)	(1, -2)	(2, -4)	(3, -6)



- From the graph, we can describe the relationship between the  $x$  values and the  $y$  values.
  - Because a straight line can be drawn through each point, we can say that there is a **linear** relationship in the data.
  - The graph goes 'downhill' from left to right, which implies the relationship is negative.

So we can deduce that the relationship is a **negative linear relationship**.

### Summary:

Positive Linear Relation	Negative Linear Relationship	Non Linear Relationship

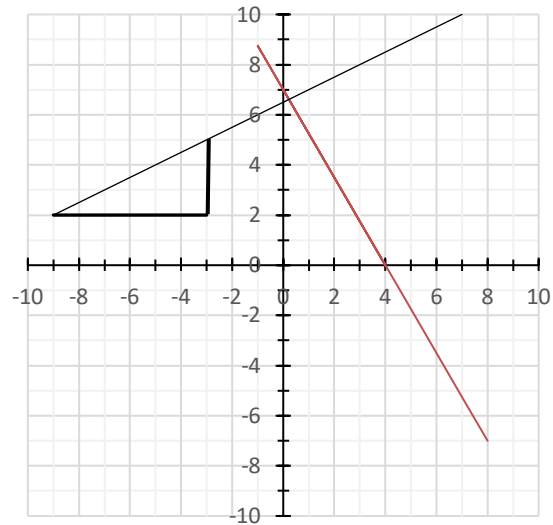
Looking at the two lines plotted on the graph below, we can see that one line is steeper than the other; this steepness is called gradient and has the symbol  $m$ .

The formula to calculate gradient is:

$$\text{gradient} = \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}}$$

$$\therefore m = \frac{\Delta y}{\Delta x}$$

Often simplified to:  $m = \frac{\text{Rise}}{\text{Run}}$



To use the formula we draw a right angled triangle on the line (trying to use easy values to work with).

On this triangle we have marked two  $y$  coordinates at 5 and 2; and two  $x$  coordinates at  $-3$ ,  $-9$ .

Thus, looking at the coordinates marked by the triangle, we can say that there is a:

- **rise** of 3: *difference in  $y$  coordinates* ( $5 - 2 = 3$ ) and
- **run** of 6: *difference in  $x$  coordinates*  $-3 - (-9) = 6$


$$\text{so } m = \frac{\text{Rise}}{\text{Run}} = \frac{3}{6} = \frac{1}{2} = 0.5$$

$\therefore$  The solid line has a positive gradient of 0.5. (It is positive because it is 'uphill'.)

### Question 19:

a) Calculate the gradient of the red line.

$$\text{gradient} = \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}}$$



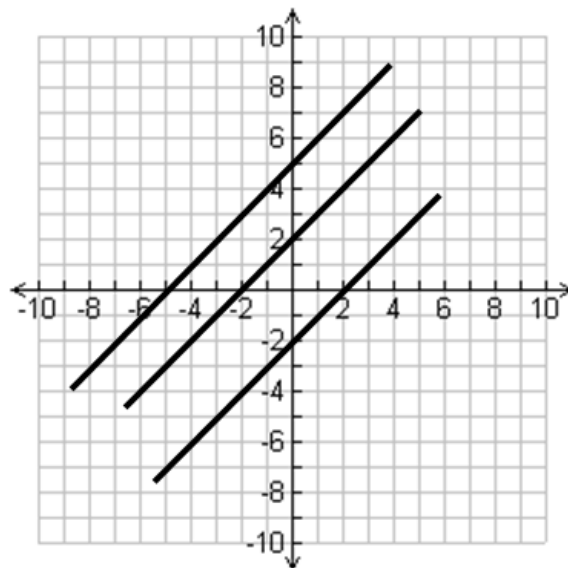
Watch this short Khan Academy video for further explanation:

**"Slope of a line"**

<https://www.khanacademy.org/math/algebra/linear-equations-and-inequalities/slope-and-intercepts/v/slope-of-a-line>

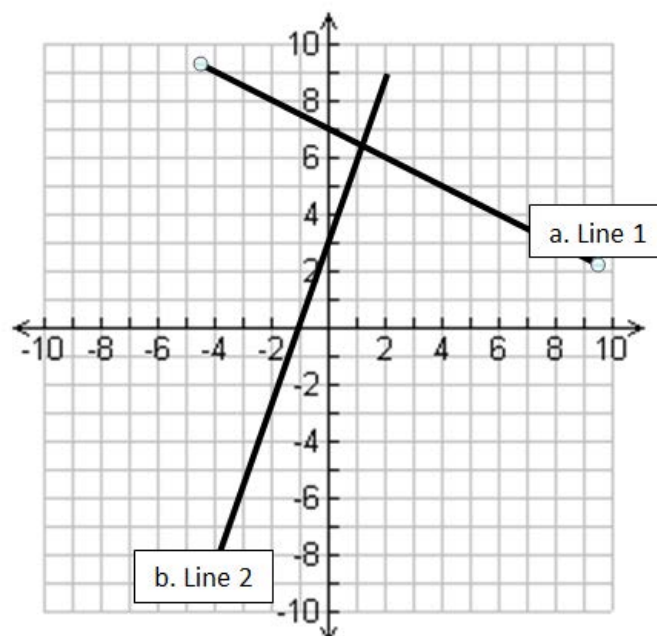
## Intercept


- Look at the three lines below, they each have the same slope, yet they are obviously different graphs. To describe how the lines differ from each other, we describe the  $y$  intercept, which is given the symbol  $c$ .
- The  $y$  intercept is the point where the line passes through the  $y$  axis. The lines below have  $y$  intercepts of 5, 2 and  $-2$ .



### Question 20:

Calculate the gradient and  $y$  intercept for the two lines, a and b, below.



 For further explanation click this link for Khan Academy:  
<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:linear-equations-graphs>

Now that we can calculate the gradient and  $y$  intercept, we can show the equation for a linear graph. All linear graphs have the following format:

$$y = mx + c$$

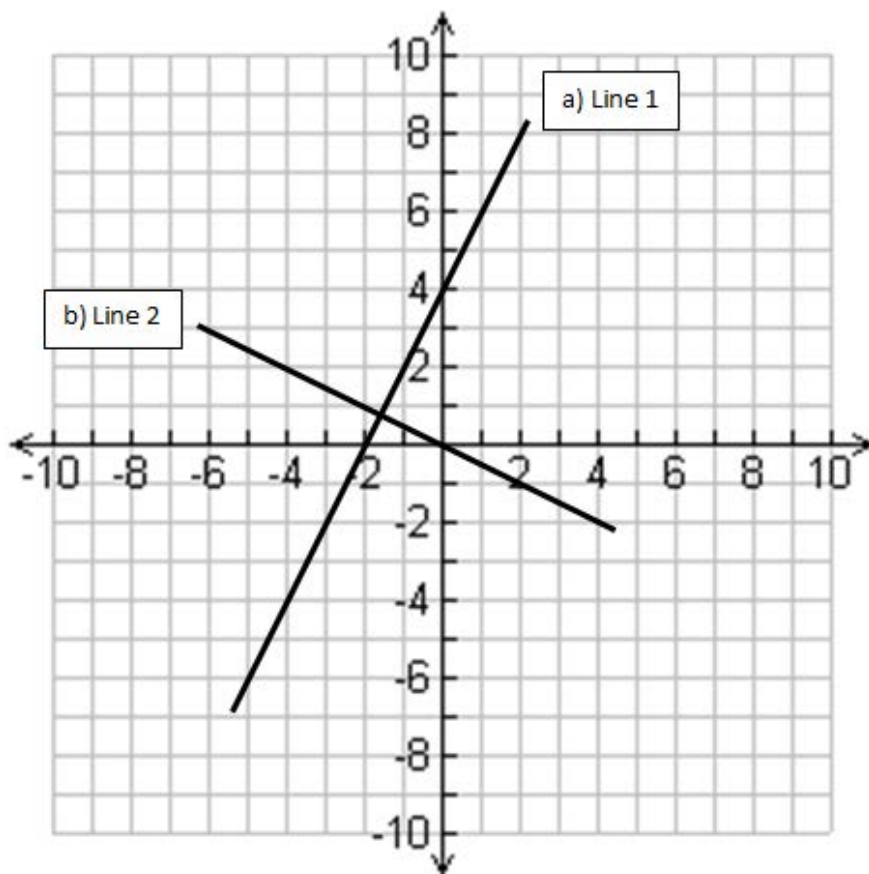
- So the equation:  $y = 2x + 3$  has a positive gradient of 2 and has a  $y$  intercept of 3.
- From Question 20, we can now show that the two lines have the equations:
  - $y = 3x + 3$
  - $y = -0.5x + 7$
- Now that we have the equation we can use it to predict points.
  - For the line  $y = 3x + 3$ , when  $x$  equals 12, we substitute  $x$  for 12, to get  $y = 3 \times 12 + 3 \therefore y$  must equal 39.

### Question 21:

Write the equation for each of the lines below and calculate the  $y$  value when  $x = 20$ .

a)

b)



Watch this short Khan Academy video for further explanation:  
"Constructing equations in slope-intercept form from graphs"

<https://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/equation-of-a-line/v/graphs->

## Graphing Equations

So far our focus has been on linear graphs, but some equations produce curved graphs.

**Example Problems:** Show the following lines on a graph and comment on the shape. Create a value table first.

1.  $y = 2x$

2.  $y = x^2$

3.  $y = \frac{1}{x}$

1.  $y = 2x$

X value	-3	-2	-1	0	1	2	3
Y value	-6	-4	-2	0	2	4	6
Coordinate	(-3, -6)	(-2, -4)	(-1, -2)	(0, 0)	(1, 2)	(2, 4)	(3, 6)

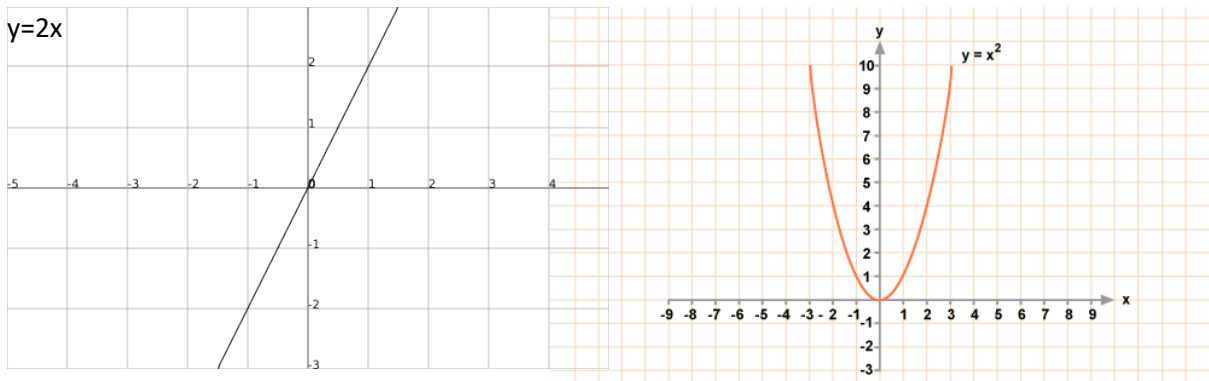
2.  $y = x^2$

X value	-3	-2	-1	0	1	2	3
Y value	9	4	1	0	1	4	9
Coordinate	(-3, 9)	(-2, 4)	(-1, 1)	(0, 0)	(1, 1)	(2, 4)	(3, 9)

3.  $y = \frac{1}{x}$

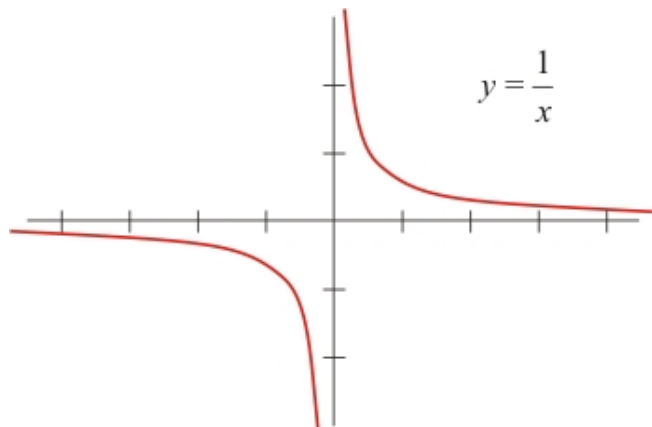
X value	-3	-2	-1	0	1	2	3
Y value	-0.33	-0.5	-1	error	1	.5	.33
Coordinate	(-3, -0.33)	(-2, -0.5)	(-1, -1)		(1, 1)	(2, .5)	(3, .33)

Step 2: Plot the graph

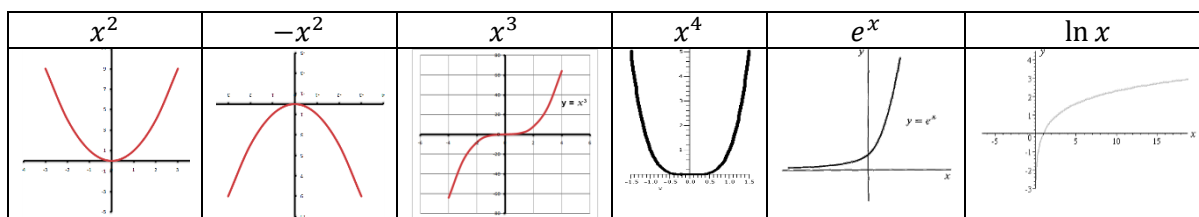


Step 3: Comment on the shape

1. Positive Linear
2. Non Linear (Parabola)
3. Non Linear (does not touch zero)



**Common curve shapes:**



**Question 22:**

On the graph below, plot the following three equations and comment on the shape.

- a)  $y = 3x - 2$
- b)  $y = x^2 + 1$
- c)  $y = -x^2 + 4$

a)

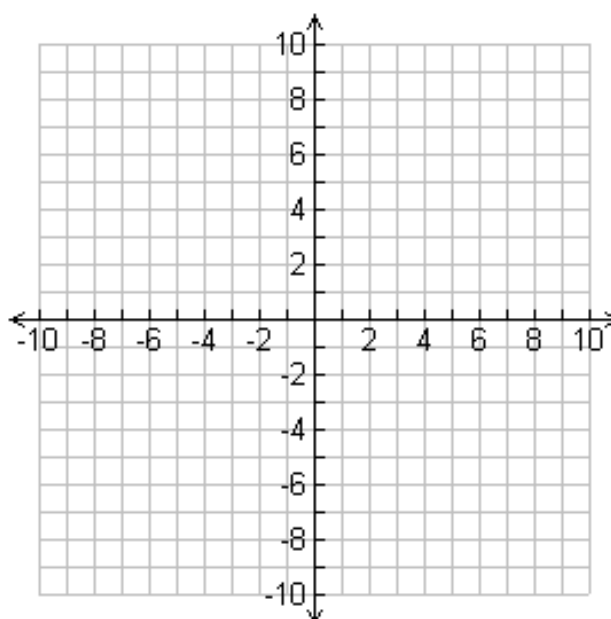
X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							

b)

X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							

c)

X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							



Watch this short Khan Academy video for further explanation:  
**"Graphs of linear equations"**

[https://www.khanacademy.org/math/algebra/linear-equations-and-inequality/graphing\\_solutions2/v/graphs-of-linear-equations](https://www.khanacademy.org/math/algebra/linear-equations-and-inequality/graphing_solutions2/v/graphs-of-linear-equations)

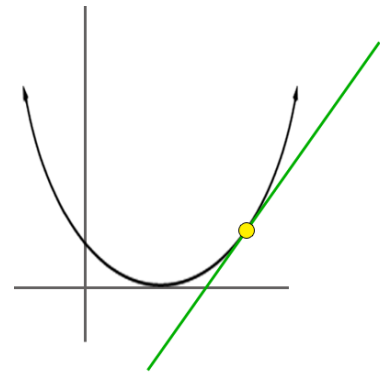
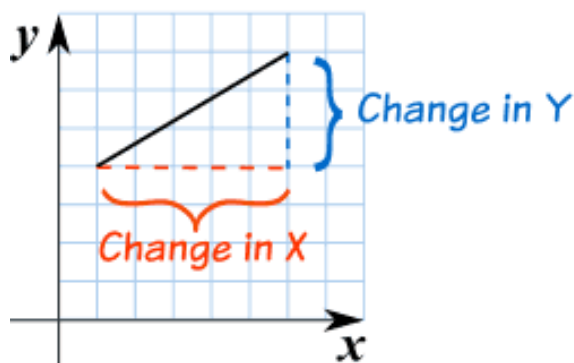


## The Gradient of a Curve

In the previous section when calculating the gradient of a straight line, we used the formula  $m = \frac{\Delta y}{\Delta x}$ . Hence, at any point on the line, the gradient remains the same. The example below left, always has a gradient of +0.6.

A curve is different; the slope of a curve will vary at different points. One way to measure the slope of a curve at any given point is to draw a **tangent**. The tangent is a straight line that will just touch the curve at the point to be measured. An example is shown in the image below right; the tangent drawn shows the gradient at that point. If the point was moved either left or right, then the angle of the tangent changes.

- The gradient of the tangent at that point equals the gradient of the curve at that point.



**Example problem:** Let's use the information from 18. b. and the plot you have drawn.

1. Plot a graph for  $y = x^2 + 1$ , for values of  $x$  between  $-3$  and  $3$ ; hence, we created a table of values:

X value	-3	-2	-1	0	1	2	3
Y value	10	5	2	1	2	5	10
Coordinate	-3,10	-2,5	-1,2	0,1	1,2	2,5	3,10

2. Draw in a tangent A at  $(1,2)$

3. Calculate the gradient of the tangent (drawn by eye at this stage) and find the gradient of the curve at A.

Thus we select two points on the tangent which are  $(-1, -2)$  and  $(-3, -6)$

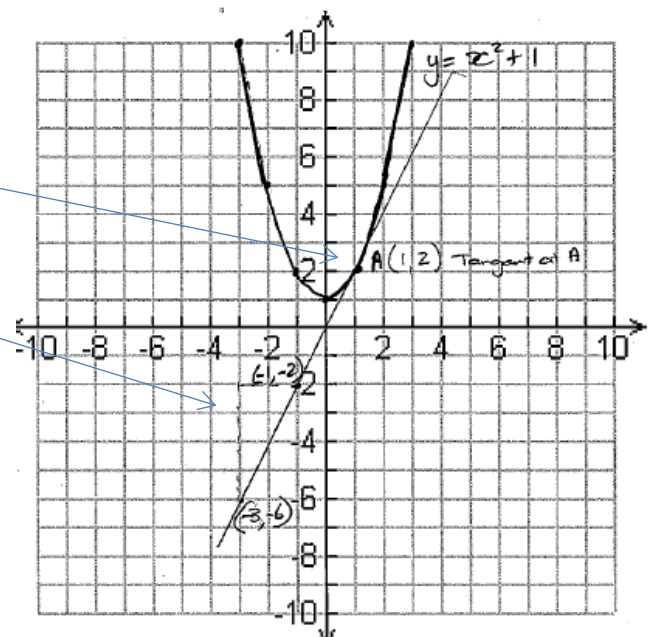
Now calculate the gradient of the tangent:  $m = \frac{\Delta y}{\Delta x}$

which is:  $gradient = \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}}$

$m = \frac{-2 - (-6)}{-1 - (-3)} = 2 \therefore$  the gradient of the tangent is 2 and thus the gradient of the curve at point A  $(1,2)$  is 2.

### Question 23:

Using the information above, calculate the gradient of a tangent at B  $(-2,5)$ .



# Answers

---

## Q 1. Order of Operations

- a) 1
- b) 22
- c) 10
- d) 12
- e) ÷

## Q2. Fraction Multiplication and Division

- 1.  $\frac{5}{12}$
- 2.  $\frac{14}{39}$
- 3.  $\frac{7}{27}$
- 4.  $1\frac{1}{14}$
- 5.  $\frac{108}{175}$
- 6.  $-\frac{1}{2}$
- 7.  $\frac{63}{8}$  or  $7\frac{7}{8}$
- 8. 4
- 9. 22 out of 25
- 10. a) 4 g    b) 2 g

## Q3. Converting Decimals into Fractions

- a)  $\frac{65}{100} = \frac{13}{20}$
- b)  $2\frac{666}{1000} \approx 2\frac{2}{3}$
- c)  $\frac{54}{100} = \frac{27}{50}$
- d)  $3\frac{14}{100} = 3\frac{7}{50}$
- e)  $\frac{40}{5} = 8$

## Q4. Converting Fractions into Decimals

- a. 0.739
- b. 0.069
- c. 56.667
- d. 5.8

## Q5. Percentage

- a. 88c
- b.. Quartz: 84.7% Feldspar: 5.7% Lithic fragment: 9.7%
- c.  $\frac{16}{25}$

## Q6. Ratios

- a) 100 mL
- b) 9 cups
- c) 8:28

## Q7. Ratio scale

- a) 3 m wide
- b) 20 cm
- c) 300 m
- d) 10 cm

## Q8. Whole ratio and proportion

- a) 240 minutes or 6 hours
- b) 9 m
- b) 4500 years; 4.5 ka (thousand years before present)

Q9. Algebra

- a) i.  $x = 13$   
 ii.  $x = 9$   
 iii.  $x(x - 3x^2 + 2)$   
 iv. 4
- b)  $6y^2x + 14y^3 + 4y^2$   
 c) 34  
 d)  $6.37 \times 10^6$  m

Q10. Power Operations

- a) i.  $5^2 \times 5^4 + 5^2 = 5^6 + 5^2$   
 ii.  $x^2 \times x^5 = x^7$   
 iii.  $4^2 \times t^3 \div 4^2 = t^3$   
 iv.  $(5^4)^3 = 5^{12}$   
 v.  $\frac{2^4 3^6}{3^4} = 2^4 3^2 = 16 \times 9 = 144$   
 vi.  $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{27}$   
 vii.  $\frac{9(x^2)^3}{3xy^2} = \frac{9x^6}{3xy^2} = \frac{3x^5}{y^2}$   
 viii.  $a^{-1}\sqrt{a} = a^{-1} \times a^{\frac{1}{2}} = a^{-\frac{1}{2}}$   
 $= \frac{1}{\sqrt{a}}$  or  $\frac{1}{a^{\frac{1}{2}}}$
- b) i.  $x = 2$   
 ii.  $x = -2$   
 iii.  $x = 3$   
 iv.  $x = 5$   
 v. Show that  $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$
- $$\frac{16a^2b^3}{3a^3b} \times \frac{9a^3b^5}{8b^2a}$$
- $$= \frac{2a^5b^8 \times 3}{b^3a^4} = \frac{2a^1b^5 \times 3}{1} = 6ab^5$$

Q11. Scientific Notation

- a)  $4.5 \times 10^2$   
 b)  $9.0 \times 10^7$   
 c)  $3.5 \times 10^0$   
 d)  $9.75 \times 10^{-2}$
- e) 375  
 f) 39.7  
 g) 0.1875  
 h) 0.00875

Q12. Calculations

- a)  $1.5 \times 10^{-5}$   
 b)  $3.375 \times 10^9$   
 c)  $-8.214 \times 10^{14}$   
 d)  $8.8612 \times 10^{10}$
- e)  $3.076 \times 10^{-2}$   
 f)  $2.126 \times 10^{-2}$   
 g) 500 s = 8.3 mins  
 h) 2058 million years

Q13. Unit Conversion

- a) 250mg km  
 b) 0.015 mg
- c) 0.067L  
 d)  $1.5 \times 10^6$  mm<sup>2</sup>

Q14 More examples

- a)  $1.2 \times 10^5$  mg  
 b)  $4.264 \times 10^{-3}$  kL & 4264 mL  
 c)  $6.7 \times 10^{-4}$  g
- d)  $1 \times 10^6$  mL  
 e) 15 inches  
 f) 114 000 cm  
 g) 1600 million years

Q15. Logarithms

- |                     |         |
|---------------------|---------|
| a) $\log_5 25 = 2$  | f) 4.03 |
| b) $\log_6 36 = 2$  | g) 4.61 |
| c) $\log_3 243 = 5$ | h) 1.48 |
| d) 4                | i) -0.3 |
| e) 2.54             | j) 0.60 |

Q16 Using Pythagorean theorem

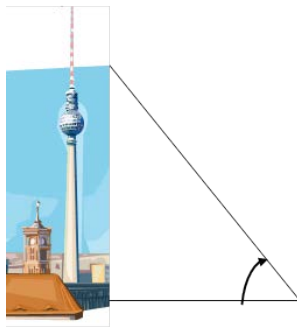
- |                                  |  |
|----------------------------------|--|
| a). 17                           | e). $\triangle ABC$ is a right angle triangle because $21^2 + 20^2 = 29^2$ |
| b) 7.94                          | whereas $\triangle DEF$ is not because $7^2 + 24^2 \neq 27^2$              |
| c) 25m                           | f) $\angle A (\angle BAC)$ is a right angle                                |
| d) i) 6cm    ii) $24\text{cm}^2$ |  |

Q17 Trigonometric functions

- a)  $\tan a = \frac{8}{15}$      $a \approx 28^\circ$     Note: to calculate the angle on your calculator, press "Shift" – " $\tan(\frac{8}{15})$ "
- $\tan b = \frac{15}{815}$      $b \approx 62^\circ$
- b)  $\sin 60^\circ = \frac{c}{10}$      $c \approx 8.66$
- c)  $\cos 28^\circ = \frac{x}{55}$      $x \approx 48.56$

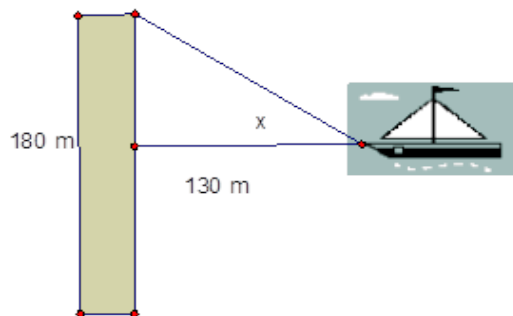
Q18 Solving trigonometric problems

a)



$$\tan 40^\circ = \frac{h}{200\text{m}} \quad h \approx 167.82\text{m}$$

b)



$$\tan x = \frac{90\text{m}}{130\text{m}} \quad x \approx 34.70^\circ$$

c)  $\frac{opp}{hyp} = \sin dip \rightarrow \frac{Tt}{30 m} = \sin 27^\circ$

$Tt = 30 m \times \sin 27^\circ$

$Tt = 13.6 m$

Q19. Gradient

a.  $-\frac{7}{4} = -1\frac{3}{4} = -1.75$

Q20. Intercept

a. Line 1:  $m = -0.5, c = 7$     b. Line 2:  $m = 3, c = 3$

Q21. Linear Equation

a. Line 1:  $y = 2x + 4 \therefore y = 44 \text{ when } x = 20$

b. Line 2:  $y = -\frac{1}{2}x + 0 \therefore y = -10 \text{ when } x = 20$

Q22. Graphing Equations

a. Positive Linear    b. U Shaped    c.  $\cap$  Shaped

Q23. The Gradient of a Curve

a.  $-4$

