Maths Refresher

Averages and Percentages

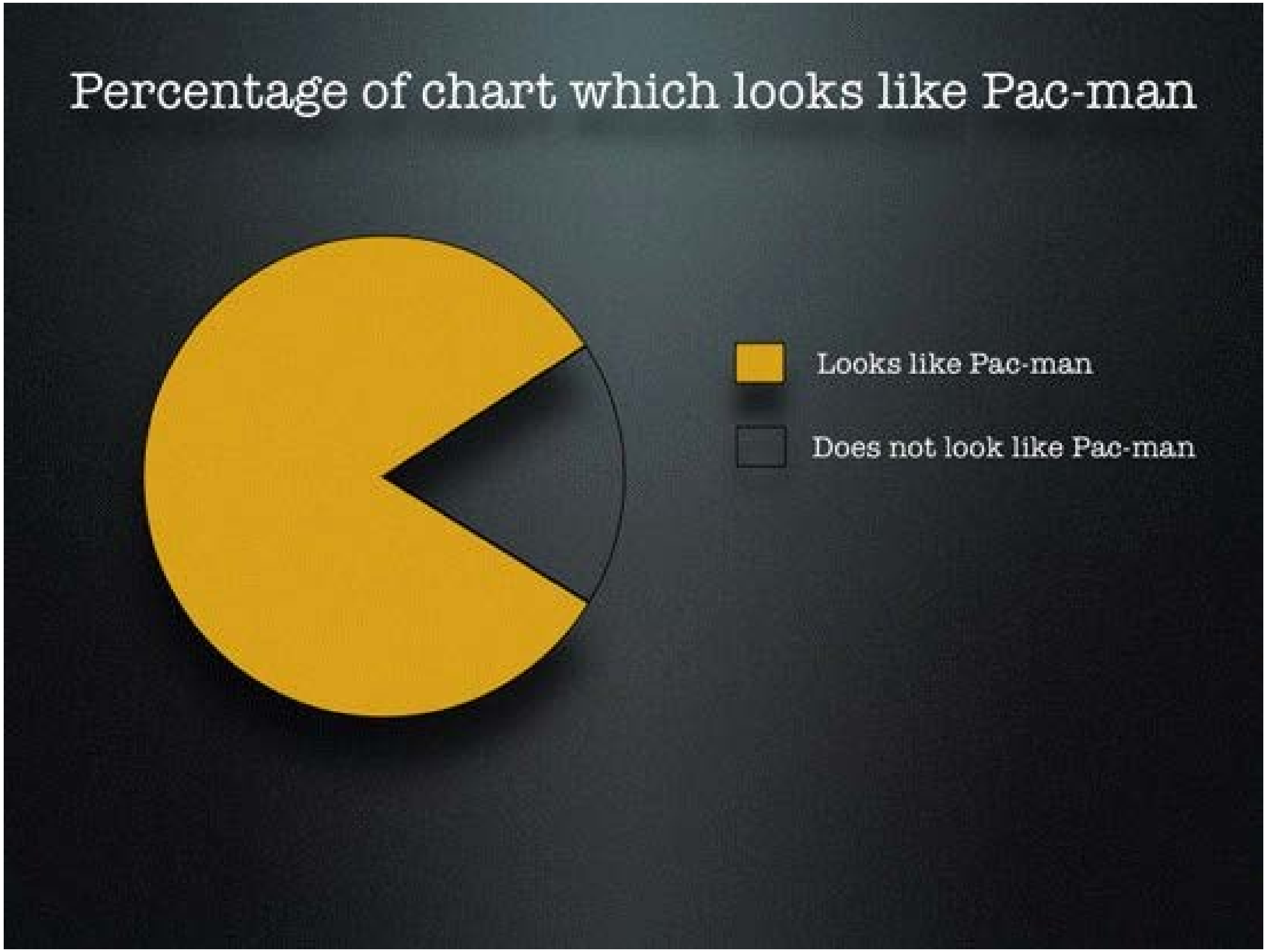
Learning, Teaching
and Student Engagement

Working with averages and percentages

Learning intentions ….

- Percentage as a fraction
- Percentage of an amount
- Using percentage to find:
  - a discounted value
  - an increased value
  - difference
- Average
- Measures of central tendency
  - mean
  - median
  - mode

Percentage


Percentage of chart which looks like Pac-man
Looks like Pac-man
Does not look like Pac-man

Percentage as a fraction

- Percentage and fractions are related.
- To allow easy comparisons of fractions we can use percentages which are simply fractions with a denominator of 100.
- The ‘whole’ is divided into 100 equal parts.
- Another way to think about per cent is that “per cent” means per 100.
- Therefore, 27% is $\frac{27}{100}$. To give 110% would be $\frac{110}{100}$!

Percentage as a fraction

- To use percentage in a calculation, simply write the percentage as a fraction with a denominator of 100.
- For example: 25% is $\frac{25}{100}$
- Thus, to calculate 25% of 40 we could approach it as $\frac{25}{100} \times \frac{40}{1} = 10$ (remember last week we talked about ‘of’ for multiplication of fractions)

Watch this short Khan Academy video for further explanation:
“Percent word problem”
https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/percent-intro-pre-alg/v/percent-word-problems

Your turn…

PERCENT – making percentages into fractions

Example problems:

1. $20\%\ into\ a\ fraction\ \frac{20}{100} = \frac{2}{10} = \frac{1}{5}\ \therefore 20\%\ is\ the\ same\ as\ \frac{1}{5}$
2. $16\%\ into\ a\ fraction\ \frac{16}{100} = \frac{4}{25}$

Practise problems:

1. $35\% = \frac{\square}{\square}$
2. $88\% = \frac{\square}{\square}$
3. $65\% = \frac{\square}{\square}$
4. $12\% = \frac{\square}{\square}$

Answers

1. $35\% = \frac{7}{20}$

3. $65\% = \frac{13}{20}$

2. $88\% = \frac{22}{25}$

4. $12\% = \frac{3}{25}$

Your turn to practise

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PERCENT – making fractions into percentages

Example problems:

1. $\frac{3}{5} \times \frac{100}{1}$

Answer: we can cancel by dividing top and bottom by 5

So, the new have 3x20= 60, $\therefore \frac{3}{5} = 60\%$

2. $\frac{5}{8} \times \frac{100}{1}$

Answer: we can cancel by dividing top and bottom by 4

So, $\frac{5}{2} \times \frac{25}{1} = \frac{125}{2} = 62.5$ $\therefore \frac{5}{8} = 62.5\%$

Practise problems:

1. $\frac{2}{5}$ *as a percent*
2. $\frac{2}{3}$ *as a percent*
3. $\frac{9}{9}$ *as a percent*
4. $\frac{4}{5}$ *as a percent*

Answers

1. $\frac{2}{5}\ as\ a\ percent = \frac{2}{5} \times \frac{100}{1}$ = 40%
2. $\frac{2}{3}\ as\ a\ percent$ = 66.6%
3. $\frac{9}{9}\ as\ a\ percent$= 100%
4. $\frac{4}{5}\ as\ a\ percent$ = 80%

Percentage of an amount

- Percentages are most commonly used to compare parts of an original.
  - For instance, a sign that reads, “Spotlight has a 30% off sale!” means that whatever the original price, the new price will be 30% (**of the original price)** less.
- Very rarely will percentages be as simple as 23% of 60.
- It is far more common for percentage calculations to be part of a more complex question, for example, “How much is left?” or “How much was the original?”

Different methods

Example problem:

I am purchasing a new iPod for $300.
At present they are reduced by 15%
How much will my iPod cost?

Method one

Method one:

- If for every $100.00 spent I save $15.00,
- There are three lots of 100 in 300,
- so 15×3 is 45,
  - think (10×3) + (5×3)

- Solution:
  Step 1: Calculate the percentage
- 15% of 300 is $45.00

  Step 2: Calculate the difference
- $300 - 45 = 255$ $\therefore$ the iPod will cost $255.00

Method Two

Method two:

Step 1: Calculate the percentage

- If 15% is the same $\frac{15}{100}$ which is also the same as 0.15, then $300 \times 0.15\ is\ 45$

Step 2: Calculate the difference

- 300 – 45 is 255, ∴ the iPod will cost $255.00

Method Three

<u>Method three:</u>

Step 1: Calculate the percentage

- If 15% is 10% + 5%, then 10% of 300 is 30.
- Then, 5% is half of 10%,
- So half of 30 is 15 (5% of 300)
- So 30+15 is 45.

Step 2: Calculate the difference

- 300 – 45 = 255 ∴ the iPod will cost $255.00

Method Four

<u>Method four:</u>

Step 1: Calculate the percentage

- 15% of 300: $\frac{15}{100} \times \frac{300}{1} = \frac{4500}{100} = 45$

Step 2: Calculate the difference

- Since we are calculating the discounted amount, we subtract 45 from the original price.
- 300 – 45 =255

Your turn …

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Which method would you use?

The local electronics shop is having a 20% off everything sale. I would like to purchase a new television that was originally priced at $600. How much will the television cost while it is on sale?

Answers

Problem: The local electronics shop is having a 20% off everything sale. I would like to purchase a new television that was originally priced at $600. How much will the television cost while it is on sale?

Method 1	Method 2	Method 3	Method 4
For every $100 I spend, I save $20. Then, there are six hundreds in 600, so: 20 x 6 = 120 ∴ Step 1: simple percentage: 20% of $600 is $120 Step 2: difference $600 - $120 = $480	20% is the same as 0.20 Then, 600 x 0.2 = 120 ∴ Step 1: simple percentage: 20% of $600 is $120 Step 2: difference $600 - $120 = $480	20% is the same as 10% + 10%. 10% of 600 is 60. 20% = 60 + 60 = 120 ∴ Step 1: simple percentage: 20% of $600 is $120 Step 2: difference $600 - $120 = $480	Step 1: SIMPLE PERCENTAGE $\frac{20}{100} \times \frac{600}{1} = \frac{12000}{100} = 120$ Step 2: DIFFERENCE Since we are calculating the discounted amount, we subtract 120 from the original price. $600 – $120 = $480

How do you find a discounted value?

METHOD A:

- An advertisement at the chicken shop states that on Tuesday everything is 22% off.
- If chicken breasts are normally \$9.99 per kilo. What is the new per kilo price?

Answer:

Step 1: Calculate the percentage

$\frac{22}{100} \times \frac{9.99}{1} = 2.20$ $\left(\frac{22}{100} \times \frac{10}{1} = 2.20\right)$

Step 2: Calculate the difference

Since the price is cheaper the 2.20 is subtracted from the original.

9.99 – 2.20 = \$7.79

How do you find a discounted value?

METHOD B:

- An advertisement at the chicken shop states that on Tuesday everything is 22% off.
- If chicken breasts are normally $9.99 per kilo. What is the new per kilo price?

$100\% - 22\% = 78\% \text{ or } 0.78$

$so\ 9.99 \times 0.78 = 7.79$

$\therefore$ The discounted price per kilo is $7.79

What if it is ***not*** a discount ?

- For the new financial year you have been given an automatic 5% pay rise.
- If you were originally on \$17.60 per hour, what is your new rate?

Step 1: Calculate the percentage

- $\frac{5}{100} \times \frac{17.6}{1} = \frac{88}{100} = 0.88$

Step 2: Calculate the difference

- Since it is a pay ***rise*** the 0.88 is ***added*** to the original.
- \$17.60 + \$0.88 = \$18.48 per hour

What if it is ***not*** a discount ?

- Or, we could simplify the previous two-step calculation into one step:

- If your new salary is the existing salary (100%) plus the increase (5%), you could calculate your new salary by finding 105% of the existing salary.

- Remembering that 105% is 1.05 as a decimal, we simply multiply 17.60 by 1.05
- So, $17.60 \times 1.05 = 18.48$
- ∴ my new salary will be \$18.48 per hour

Find the percentage difference?

- A new dress is now $236 reduced from $400. What is the percentage difference?
- As you can see, the problem is in reverse, so we approach it in reverse…

Step 1: Calculate the difference

Since it is a discount the difference between the two is the discount.

400 – 236 = 164

Step 2: Calculate the percentage

$$\frac{?}{100} \times \frac{400}{1} = 164$$

….*continued on next slide*

Find the percentage difference?

…*continued*

Working mathematically: rearrange the problem in steps:

$$\frac{x}{100} = \frac{164}{400}$$ what we do to one side we do to the other

$$\frac{x}{100} \times \frac{100}{1} = \frac{164}{400} \times \frac{100}{1}$$ (we multiply both sides by 100)

$$x = \frac{16400}{400}$$ (simplify: divide both top & bottom by 100)

$x = \frac{164}{4} = 41$, so:

∴ the percentage difference is 41%

Find the original value ?

You can also use this approach to calculate an original value:

- A clothing shop pays $300 wholesale for a shirt, to make a profit the cost is marked up by 25%, resulting in a retail price of $375.
- $300 \times 1.25 = 375$ (using the one step method from above)

Now let's get back to the original cost.

- If $375 now represents 125% , what is 100%?

$$\frac{375}{125} \times \frac{100}{1} \ (\textit{cancel down: divide the top and bottom by } 25)$$

$$\frac{15}{5} \times \frac{100}{1} \ (\textit{then divide top and bottom by } 5)$$

$$\frac{3}{1} \times \frac{100}{1} = 300$$

So, the wholesale cost was $300

Your turn …

Interpreting word problems

Read each question carefully; decide what it is that you need to do – use the examples from the PowerPoint as a guide.

1. I want to buy some shoes that are reduced by 15%. Their original price is marked at $200. How much will a pay for the discounted shoes?
2. You have received a 6% increase on your weekly pay. If you originally received $512 per week, what is your new weekly wage?
3. In 2012, an ornithologist conducted a survey of brolgas sighted in a nearby wetland. She counted 24 individuals. In 2013, she counted 30 individuals. By what percentage did the population of brolgas increase?
4. Fiona scored 24 out 30 on her first maths test. On her second test, she scored 22 out of 30. By what percentage did Fiona's maths results change?
5. A computer screen was $299 but is on special for $249. What is the percentage discount?

Answers

1. First, we can deduct 15% from 100% giving us 85%. 85% as a decimal is 0.85

So, we simply use 200 x 0.85 = 170

Therefore, our new discounted price for the shoes is $170.

2. $512 x 1.06 = $542.72 per week

3. $\frac{?}{100} \times \frac{24}{1} = 6;\ \frac{x}{100} = \frac{6}{24}$ We divided both sides by 24

$x = \frac{6}{24} \text{ x } \frac{100}{1}$ We multiply both sides by 100

$x = \frac{600}{24}$; x = 25%

Answers

4. Score 1 = $\frac{24}{30}$ = 80%; Score 2 = $\frac{22}{30}$ = 73.3%

Therefore, Fiona's score decreased (80 – 73.3) 6.7%

5. 299—249 = 50 we work out the difference first

$\frac{50}{299} \times \frac{100}{1}$ = 16.7%

GST

Now for something a bit trickier: What if we were to add 10% GST to an item that costs $5.50. We can do this in one step, again converting percentage to decimals: $5.50 \times 1.10 = \$6.05$

- Now tax time comes and we want to get back to the original cost. Can we simply subtract 10%?
- $\$6.05 \times 0.9 = \$5.44$ which is not $5.50; so what do we do?
- We need to think that if 10% was added, then $6.05 is 110% because we have the whole 100% + 10% so…
- To calculate we write: if $6.05 is 110% $what\ was\ 100\%$
  - $\frac{6.05}{110} \times \frac{100}{1} = \frac{605}{110} = 5.5$
- Now we are back to the original cost of $5.50

Your turn …

GST adds 10% to the price of most things. How much does a can of soft drink cost if it is 80c before GST?

Answer

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If a soft drink cost 88c with GST, how much did it cost before GST?

$0.80

$$\frac{0.88}{110} \times \frac{100}{1} = \frac{88}{110} = 0.80$$

Challenge …

- If 3 litres of cordial are poured into a 5 litre jug, what percentage of the jug is full?

Challenge answer

$$\frac{3}{5} \times \frac{20}{20} = \frac{60}{100} = 60\%$$

“Average”

- The average Sydney-sider

A photographer has compiled photos of people from cities around the world in an attempt to determine what the average human face looks like.

The photographer creates his final "average" face by photographing the first 100 people who concur to sit for his project in each city. He then takes the images and combines them digitally to make a new face.

Today World News
Thursday, February 10, 2011

Measures of central tendency

When examining a collection of data, it is most common to find a measure of the "middle" value. The three common measures of central tendency are:

- Mean
- Median
- Mode

"Statistics intro: Mean, median, mode"

https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-central-tendency/v/statistics-intro-mean-median-and-mode

Measures of central tendency

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- **Mean:** The most commonly used measure. Often referred to as the average. It is calculated by adding up all the data and dividing by how many pieces of data you had. For example, the average temperature over one month might be calculated using mean.
- The formula for mean is: $\bar{x} = \frac{\Sigma x}{n}$
- $\bar{x}$ is called "x bar" and represents mean.
- Σ is called "sigma" and means sum or add up
- x is the individual pieces of data
- n is how many pieces of data

Other measures of central tendency …

- **Median:** The middle number when the data is arranged in order. Relating to the 'median strip' in the road is one way to remember what type of central tendency median implies.

- **Mode:** The most common number. We can have more than 1 mode such as bimodal (2 modes). Mode is useful in situations where the most common size is needed to determine the most appropriate equipment to purchase. For example, to purchase chairs or hats, the most common height or head size would be used.

Mean

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Example Problem 1:

- A girl playing 5 games of cricket scores 35 runs in the first game, in the 2nd, 140 in the 3rd, 50 in the 4th, and 35 in the last game.
- We want to show her batting average to make her look the best.

35 27 140 50 35

- **Mean** – add the totals and divide by the total number of events (games).

35+27+140+50+35= 287

287÷5=57.4

- Therefore her batting average is 57.4 or 57 runs a game.

Median

- **Median** – line up the numbers from smallest to largest and work towards the middle number.

- 27 35 35 50 140

  → → 35 ← ←

- Therefore, her median batting score is 35

Mode

- **Mode** – the number that occurs most common in the chain of events
- 35   27 140   50 35
- Therefore, her most frequent batting score is 35
- If we wanted to show the batting average that might make this girl look the best she can be, then we would choose the 'mean average.' The mean average shows that the girl has an average of 57. In this case the **outlier** score (140) has skewed the data, resulting in a mean that is greater than either the median or modal score.

Your turn….

The following is a data set that records one player's point score in seven consecutive games of basketball.

30, 80, 56, 22, 22, 36, 39

- Calculate the three different measures of central tendency:

1. Mean:
2. Median:
3. Mode:
4. Which measure of central tendency is the most meaningful?

Answers:

1. 41
2. 36
3. 22
4. mean (depending on the purpose, who, what, why)

Working with averages and percentages

Reflect on the learning intentions ….

- Percentage as a fraction
- Percentage of an amount
- Using percentage to find:
  - a discounted value
  - an increased value
  - difference
- Average
- Measures of central tendency
  - mean
  - median
  - mode