

Learning, Teaching  
and Student Engagement

# Maths Module 1

## An Introduction to Mathematics

This module covers concepts such as:

- basic arithmetic
- rounding
- order of operations
- mental computation strategies



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# Module 1

## An Introduction to Mathematics

1. Introduction to Arithmetic
  2. Arithmetic of Whole Numbers
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# 1. Introduction to Arithmetic

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- Mathematics is the science of patterns and relationships related to quantity. For example, below are some relationships of the many you may have come across:
  - Every number is related to every other number in a number relationship. For example, 8 is 2 less than 10; made up of 4 and 4 (or 3 and 5); and is 10 times 0.8; is the square root of 64; and so on....
  - Number relationships are the foundation of strategies that help us remember number facts. For instance, knowing  $4 + 4 = 8$  allows one to quickly work out  $4 + 5 = 9$  (one more than 8); If one knows that  $2 \times 5 = 10$ , then  $4 \times 5$  and  $8 \times 5$  can easily be calculated (double 2 is 4 and so double 10 is 20; then double 4 is 8 and so double 20 is 40).
  - Each digit in a written numeral has a 'place' value which shows its relationship to '1'. For example, in 23.05 the value of the '2' is 20 ones, while the value of the '5' is only five-hundredths of one. Understanding place value is critical when working with numbers.
- Mathematics is considered a universal language; however, words in English can often have more than one meaning which is why we sometimes find it difficult to translate from English to mathematical expressions.
- Arithmetic is a study of numbers and their manipulation.
- The most commonly used numbers in arithmetic are integers, which are positive and negative **whole** numbers including zero. For example: -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6. Decimal fractions are not integers because they are 'parts of a whole', for instance, 0.6 is 6 tenths of a whole.
- The symbols we use between the numbers to indicate a task or relationships are the **operators**, and the table below provides a list of common operators. You may recall the phrase, 'doing an operation.'

Symbol	Meaning
+	Add, Plus, Addition, Sum
-	Minus, Take away, Subtract, Difference
×	Times, Multiply, Product,
÷	Divide, Quotient
±	Plus and Minus
a	Absolute Value (ignore negative sign)
=	Equal
≠	Not Equal
<	Less than
>	Greater than
≪	Much Less than
≫	Much More than
≈	Approximately equal to
≤	Less than or equal
≥	Greater than or equal

### EXAMPLE PROBLEMS:

1.  $12 \pm 6 = 6$  and 18.  $12 + 6$  and  $12 - 6$
2.  $|-6| = 6$
3.  $12.999 \approx 13$
4.  $12 \neq 7$
5. Drink driving is a blood alcohol level of  $\geq 0.05$
6. Speed of light  $\gg$  speed of sound

### 1. Your Turn:

Are the following statements true?

- a.  $32 \neq 4 \times 8$
- b.  $7 > 6$
- c.  $4 \geq 4$
- d.  $|5| = 5$
- e.  $37.1 + 22.02 = 59.3$

## 2. Arithmetic of Whole Numbers

### Integers

Whole numbers are integers; there are positive and negative integers. Positive integers are 1, 2, 3, 4, 5... The negative integers are ... -5, -4, -3, -2, -1 (the dots before or after the sequence indicate that there are more numbers in this sequence that continue indefinitely).

Here are some more terms for you:

An **equation** implies that what is on either side of the '=' sign balances.

The **sum** of two numbers implies two numbers are added together.

The sum of 4 and 8 is 12;  $4 + 8 = 12$

The **difference** of two numbers implies that the second number is subtracted from the first number.

The difference between 9 and three is 6;  $9 - 3 = 6$

The **product** of two numbers implies that two numbers are multiplied together.

The product of 3 and 4 is 12;  $3 \times 4 = 12$

The **quotient** of two numbers implies that the first number is divided by the second.

The quotient of 20 and 4 is 5;  $\frac{20}{4} = 5$  or  $20 \div 4 = 5$

**Rational Number:** The term rational derives from the word ratio. Hence, a rational number can be a described by a ratio of integers or as a fraction. For example,  $\frac{3}{4}$  and 0.75 are both rational numbers.

**Irrational number:** A number that cannot be written as a simple fraction or as a decimal fraction. If the number goes on forever without **terminating**, and without **repeating**, then it is an irrational number. For example,  $\pi$  is a recurring decimal that does not repeat: 3.14159... Therefore,  $\pi$  is an irrational number.

**Factors:** A whole number that divides exactly into another number.

1, 2, 3, 5, 6, 10, 15, 30 are factors of 30

**Multiple:** A number that can be divided by another number without a remainder.

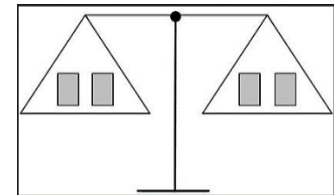
3, 6, 9, 12, 15 are all multiples of 3

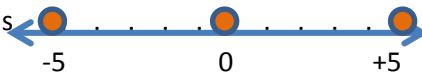
### Directed Numbers (negative and positive integers)

Directed numbers are like arrows with a particular direction  $\rightarrow +$  and  $\leftarrow -$ . They have positive and negative signs to signify their direction. Note that when using the calculator we use the  $(-)$  key rather than the subtraction key. Each negative number may need to be bracketted  $(-3) + (+3) = 0$ .

When naming directed numbers, we use the terms *negative* and *positive* numbers; avoiding the terms *plus* and *minus* unless you are indicating that an operation is taking place of addition and subtraction.

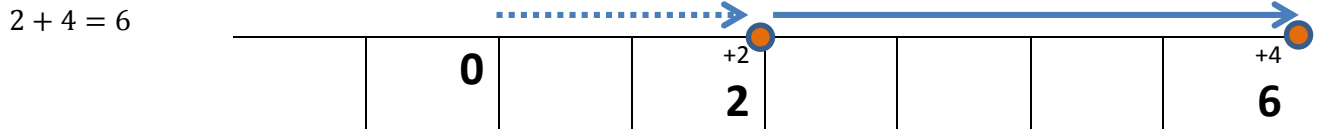
Thus  $(-3) + (+3) = 0$  reads '*negative 3 plus positive 3 equals zero*'.



To use a graphic symbol we can display  $(-5)$  and  $(+5)$  as 

This graphic symbol is known as a number line and can be used to show how and why operations work.

**Addition:** To add a number we move to the right:



In this example we added a positive number beginning at a negative number. So how it works – start a zero, move in a negative direction two places, then move in a positive direction six places; we end at four.

Your turn to represent  $0.5 + 3 =$

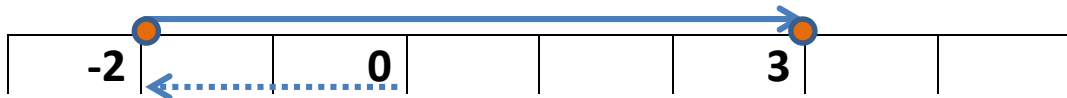


**Subtraction:**

To subtract a positive number we move that number of places to the left. For example,  $3 - 4 = (-1)$ ; we start at zero, move three spaces to the right (in a positive direction), then move four spaces to the left (in a negative direction) to end up at negative one. We can think about this as if we were on a lift. If we start at the ground floor and go up three floors, then down four floors, we would be one level below ground.

Remember that to subtract an integer means adding its opposite. Hence, to subtract a negative number we move to the right rather than the left – hence in a positive direction.

For example:  $(-2) - (-5) = 3$ ; ‘negative 2 minus negative 5’ which means  $(-2) + 5$  (adding the opposite)



Your turn to represent  $(+2) - (+5) = (-3)$



**Two key points:**

- Subtracting a negative number is the same as adding its opposite.  $4 - (-3) = 4 + 3 = 7$
- Adding a negative number is the same as subtracting a positive number.  $4 + (-2) = 4 - 2 = 2$

## 2. Your Turn

- a. Find the sum of 3, 6 & 4
- b. Find the difference of 6 and 4
- c. Find the product of 7 & 3
- d. Find the quotient of 20 and 4
- e. Find the factors of 24
- f. Find some multiples of 7

## Refreshing your memory of traditional methods for arithmetic

### Addition

1 ←

- 47 The first step is to add the ones and we get 13ones or 3ones and 1ten, so we add the 1ten to the tens

$$\begin{array}{r} + 86 \\ 47 \\ \hline 133 \end{array}$$

then we add 5tens and 8 tens to get 13tens or 130+3

### Subtraction

- $74 - 36 =$  we can read 74 as 7tens and 4ones or 6tens and 14ones. This is called **decomposing** numbers.

$$\begin{array}{r} 6 \ 14 \\ 74 \\ - 36 \\ \hline 38 \end{array}$$

so first we cannot take 6 from 4, so we decompose now we can subtract 6 ones from 14ones then we take 3 tens from 6 tens

### Division

#### Traditional bring-down method

70

$$\begin{array}{r} 5 \ 8 \\ 63 \overline{) 3742} \\ \underline{315} \\ 592 \\ \underline{504} \\ 88 \\ \underline{63} \\ 25 \end{array} = 59 \text{ R } 25$$

This is a method of long division you may recall. It was taken from the following book which covers various computation strategies. You will notice the 70 in a bubble; this reflects an easy number to work with to estimate  $374 \div 63$ .

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Sydney: Pearson Education.

### Multiplication

- Let's look at  $47 \times 65$
- Let's estimate first  $50 \times 60 = 3000$

43 ←

- 47 multiply the **ones**,  $7 \times 5 = 35$  rename 3tens and 5ones

$$\begin{array}{r} \times 65 \\ 47 \\ \hline 235 \\ 2820 \\ \hline 3055 \end{array}$$

$5 \times 4$  is 20tens, add 3tens = 23 tens,  
235 rename: 2hundreds 3tens and five ones

- 2820 multiply the **tens**, 6tens  $\times$  7ones = 42tens, 4 hundreds & 2tens
- 3055 6 tens  $\times$  4 tens are 24 hundreds plus 4 hundreds = 28 then add  $235 + 2820 = 3055$

## 2. Your Turn:

Calculate the following:

g.  $153 + 69$

h.  $132 - 78$

i.  $953 \div 18$

j.  $692 \times 36$

Sometimes when writing a product calculation, the multiplication sign 'x' is replaced with a dot '.' or '\*' or it can be omitted.

For example:

$3 \times 6 \times 9$  could be written as  $3.6.9$  or as  $(3)(6)(9)$

$3(4 + 5)$  can also be written as  $3 \times (4 + 5)$

This table will assist you multiplication and division of positive and negative numbers.

Rules for multiplying and dividing positive and negative integers	
(positive) x (negative) = negative	(positive) $\div$ (negative) = negative
(negative) x (positive) = negative	(negative) $\div$ (positive) = negative
(negative) x (negative) = positive	(negative) $\div$ (negative) = positive
(positive) x (positive) = positive	(positive) $\div$ (positive) = positive

## 2. Your Turn:

k.  $3 \times (-2) =$

l.  $(-1) \times 7 =$

m.  $(-2)(-4) =$

n.  $12 \div (-4) =$

o.  $\frac{-8}{4} =$  (tip: another way to signify division)

# 3. Working with Decimals

## Key Ideas:

The decimal separates whole numbers from parts of a whole.

For instance, 3.6; three is the whole number and 6 tenths of a whole.

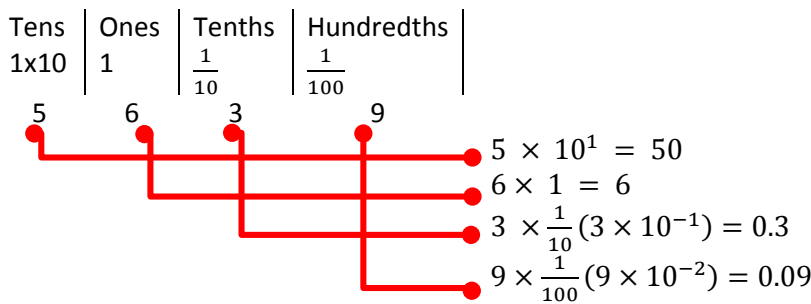
Each digit in a number has a 'place value' (related to one).

The value depends on the position of the digit in that number.

Each position can be thought of as columns.

Each column is a power of ten.

For example, let's look at 56.39



### Base Ten Relationships

The number to the right is  $\frac{1}{10}$  the amount.

↓

16,999.568

↑

The number to the left is 10 times as much!

A **recurring decimal** is a decimal fraction where a digit repeats itself indefinitely

For example, two thirds = 0.666666

Because the number repeats itself from the tenths position a dot can be written above the 6 as such  $0.\dot{6}$

If the number was one sixth ( $\frac{1}{6}$ ), which as a decimal is 0.16666, then we signify as  $0.1\dot{6}$

If the number contained a cluster of repeating digits, for example, five elevenths = 0.454545 we write  $0.4\dot{5}$

A terminating decimal is a number that terminates after a **finite** (not infinite) number of places, for example:

$$\frac{2}{5} = \frac{4}{10} \text{ or } 0.4; \text{ and } \frac{3}{16} = 0.1875 \text{ (terminating after 5)}$$

## Working with Decimals

What happens when we multiply or divide by ten, or powers of ten?

Patterns are identified when multiplying by ten. However, often it is said that we simply add a zero. Is this correct? Think about  $4.3 \times 10$ , does it equal 4.30?

Another misconception is that when multiplying by ten we move the decimal one place, whereas, it is actually the digits that move.

When we multiply by ten, all digits in the number become ten times larger and they move to the left.

What happens when we divide by 10? All the digits move to the right.

$963.32 \div 10 = 96.332$  all of the digits move one place to the right.

$963.32 \times 10 = 9633.2$  all of the digits move one place to the left.

$$\begin{array}{r} 65.320 \\ + 74.634 \\ \hline \end{array}$$

Another key point when working with addition or subtraction of numbers is to line up the decimal points. A zero can be regarded as a place holder. For example,  $65.32 + 74.634 =$  (see right) →

$$139.954$$





# 5. Order of Operations

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Look at the example problem below

$$3 + 6 \times 2 = ?$$

If I do the addition, then the multiplication, the answer would be:  $9 \times 2 = 18$

If I do the multiplication, then the addition, the answer would be:  $3 + 12 = 15$

There can NOT be two answers to the same question. A rule is required to make sure everyone uses the same order.

There is a **calculation priority sequence** to follow. Different countries, different states, even different teachers use different mnemonics to help you remember the order of operations, but common versions are BOMDAS or BIMDAS which stand for:

Brackets	{[( )]}
Other or Indices	$x^2, \sin x, \ln x, etc$
Multiplication or Division	$\times$ or $\div$
Addition or Subtraction	$+$ or $-$

## The Calculation Priority Sequence:

1. Follow the order (BOMDAS or BIMDAS)
2. If two operations are of the same level e.g. ( $\times$  or  $\div$ ) or ( $+$  or  $-$ ), you work from left to right.
3. If there are multiple brackets, then work from the inside set of brackets outwards.

### EXAMPLE PROBLEMS:

1. Solve:  $5 + 7 \times 2 + 5^2$   
Step 1:  $5^2$  has the highest priority so:  $5 + 7 \times 2 + 25$   
Step 2:  $7 \times 2$  has the next priority so:  $5 + 14 + 25$   
Step 3: only addition left so left to right:  $19 + 25$   
 $44$   
 $\therefore 5 + 7 \times 2 + 5^2 = 44$

2. Solve:  $[(3 + 7) \times 6 - 3] \times 7$   
Step 1:  $(3 + 7)$  has the highest priority:  $[10 \times 6 - 3] \times 7$   
Step 2:  $10 \times 6$  inside brackets has the next priority:  $[60 - 3] \times 7$   
Step 3: the brackets and then the multiplication:  $57 \times 7$   
 $= 399$   
 $\therefore [(3 + 7) \times 6 - 3] \times 7 = 399$

## 5. Your Turn:

a.  $4 \times (5 + 2) + 6 - 12 \div 4 =$

h.  $(200 - 150) \div 20 =$

b.  $3 \times 7 + 6 - 2 + 4 \div 2 + 7 =$

i.  $2.4 - 0.8 \times 5 + 8 \div 2 \times 6 =$

c.  $10 + 4 \times 6 =$

j.  $(2.4 - 0.8) \times 5 + 8 \times 6 \div 2 =$

d.  $(10 + 4) \times 6 =$

k.  $60 + 40 - 30 \times 2 + 8 \times 5 =$

e.  $50 - 6 \times 8 =$

l.  $60 + (40 - 30) \times (2 + 8) \times 5 =$

f.  $(50 - 6) \times 8 =$

m.  $\frac{120}{3 \times 4} =$

g.  $200 - 150 \div 20 =$

Tip: When working the examples given, use a lot of space to set your working out as a series of logical steps. This will assist your mathematical thinking and reasoning.

## 6. Answers

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1. a. False      b. True      c. True      d. True      e. False
2. a. 13      b. 2      c. 21      d. 5      e. 1, 2, 3, 4, 6, 8, 12, 24  
f. 14, 21, 28...      g. 222      h. 54      i.  $52.9\dot{4}$  or  $52r17$  or  $52\frac{17}{18}$   
j. 24912      k. -6      l. -7      m. 8  
n. -3      o. -2
3. No questions for section three
4. a. (i) 34.599      a. (ii) 56.673  
b. (i)  $\approx 35 \times 60 = 2100$       b. (ii)  $\approx 36 - 13 = 23$       b. (iii)  $\approx 30 \times 4 = 120$
5. a. 31      b. 34      c. 34      d. 84      e. 2  
f. 352      g. 192.5      h. 2.5      i. 22.4      j. 32  
k. 80      l. 560      m. 10

## 7. Helpful Websites

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- Integers: <http://www.factmonster.com/jpka/A0876848.html>
- Rounding: <http://www.mathsisfun.com/rounding-numbers.html>
- Estimating: [http://mathandreadinghelp.org/how\\_to\\_estimate\\_a\\_math\\_problem.html](http://mathandreadinghelp.org/how_to_estimate_a_math_problem.html)
- Order of Operations: [http://www.mathgoodies.com/lessons/vol7/order\\_operations.html](http://www.mathgoodies.com/lessons/vol7/order_operations.html)

## 8. Mental Computation Strategies

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On the following pages are some mental computation strategies to explore. Developing strategies to work and solve basic number facts mentally is critical for future mathematical thinking and reasoning. You may have some others that you use and may be willing to share. If so, please email any suggestions to [kerry.smith@jcu.edu.au](mailto:kerry.smith@jcu.edu.au)

The suggested strategies below have been adapted from:  
Anderson, J., Briner, A., Irons, C., Shield, M., Sparrow, L., & Steinle, V. (2007), *The Origo handbook of mathematics education* (Section 3.1). Australia: Origo Education.

# Addition



## Adding the places

See

- $35 + 23 = \underline{\quad}$

Think

- $35 + 20 + 3$

- $7 + 5 = 12$   
addend addend sum

## Bridging to ten

See

- $17 + 8 = \underline{\quad}$

Think

- $17 + 3 + 5$

## Compensating

See

- $28 + 36 = \underline{\quad}$

Think

- $(30 + 36) - 2$

## Counting on

See

- $58 + 24 = \underline{\quad}$

Think

- $58 + 10 + 10 + 4$

## Using compatible addends

See

- $14 + 23 + 16 = \underline{\quad}$

Think

- $14 + 16 + 23$   
(using compatible pairs; rainbow facts)

## Using doubles

See

- $7 + 8 = \underline{\quad}$

Think

- $7 + 8 = 15$   
*Because  $7 + 7 = 14$*

## Addition

### Using place value

See

- $56+17=$  \_\_\_

Think

- $50+10+6+7$

Or  $6+7+50+10$

- Expand the addends to places, or this can be called a 'split strategy' – breaking the numbers into more manageable bits

## Subtraction

### Adjusting

See  $83-58=$  \_\_\_

Think:

$80-55$  (subtract 3 from each)

### Bridging to ten

See  $45-12=$  \_\_\_

Think:

$45-5-7$

### compensating

See  $82-35=$  \_\_\_

Think:

$(80-35)+2$

### Counting back

See  $86-24=$  \_\_\_

Think:

$86-10-10-4$

# Subtraction

## Subtracting the places

See  $68-35=$  \_\_\_\_

Think:

$$68-30-5$$

## Thinking addition

See  $15-12$

Think:

$12+3$  is 15, so

$15-12$  is 3

## Using place value

$$12 - 7 = 5$$

Minuend   subtrahend   difference

See  $56-32=$  \_\_\_\_

Think:

$$(50-30)+(6-2)$$

Expanding the minuend and subtrahend otherwise thought as the split strategy

# multiplication

## Breaking a factor

$$\begin{array}{ccccccc} \bullet & 3 & \times & 4 & = & 12 \\ \text{Multiplicand} & & & \text{multiplier} & & \text{product} \end{array}$$

Factors

See  $26 \times 4 =$  \_\_\_\_

Think:

$$(25 \times 4) + (1 \times 4)$$

Splitting into manageable bits (not places)

# multiplication



## Building down

See  $19 \times 3 = \underline{\quad}$

Think:

$20 \times 3 = 60$ , so  
 $19 \times 3 = 57$  ( $60 - 3$ )

## Building up

See  $21 \times 3 = \underline{\quad}$

Think:

$20 \times 3 = 60$ , so  
 $21 \times 3 = 63$  ( $60 + 3$ )

## factorising

See  $15 \times 7$

Think:

$3 \times 5 \times 7$  or  $5 \times 3 \times 7$

## Recognising midpoints

See  $8 \times 15$

Think:

$8 \times 15$  must be halfway  
between  $8 \times 10$  (80) and  
 $8 \times 20$  (160)

## Doubling

See  $8 \times 14$

Think:

double 14 is 28  
double 28 is 56  
double 56 is 112

## Doubling and halving

See  $12 \times 15 = \underline{\quad}$

Think:

$6 \times 30$   
(use an array, or blocks,  
to demonstrate this  
strategy)



## multiplication

### Using a benchmark number

See  $32 \times 5 = \underline{\quad}$

Think:

$32 \times 10 = 320$ , so  $32 \times 5$  must be one half of 320 (160)

See  $16 \times 25 = \underline{\quad}$

Think:

$16 \times 100 = 1600$ , so  $16 \times 25$  must be one quarter of 1600 (400)

### Using division

See  $1/5 \times 30 = \underline{\quad}$

Think:

$30 \div 5$

### Using place value

• See  $26 \times 3 = \underline{\quad}$

Think:

$(20 \times 3) + (6 \times 3)$

## Division

### adjusting

$12 \div 3 = 4$   
Dividend   divisor   quotient

See  $75 \div 5 = \underline{\quad}$

Think:

$150 \div 10$  (double each number)

# Division



## Breaking the dividend

See  $138 \div 3 = \underline{\quad}$

Think:

$$(120 \div 3) + (18 \div 3)$$

## halving

See  $112 \div 8 = \underline{\quad}$

Think:

half of 112 is 56

half of 56 is 28

half of 28 is 14

## Thinking multiplication

See  $35 \div 7 = \underline{\quad}$

Think:

$$7 \times 5 = 35, \text{ so } 35 \div 7 \text{ is } 5$$

## Using place value

See  $318 \div 3 = \underline{\quad}$

Think:

$$(300 \div 3) + (18 \div 3)$$

# Reference



Anderson, Briner, Irons, Shield, Sparrow, Steinle (2007), *The ORIGO handbook of mathematics education*. Australia: PrintPoint