

## **Basic Statistics**

Sample vs. Population Distributions

Learning, Teaching and Student Engagement





#### Remember:

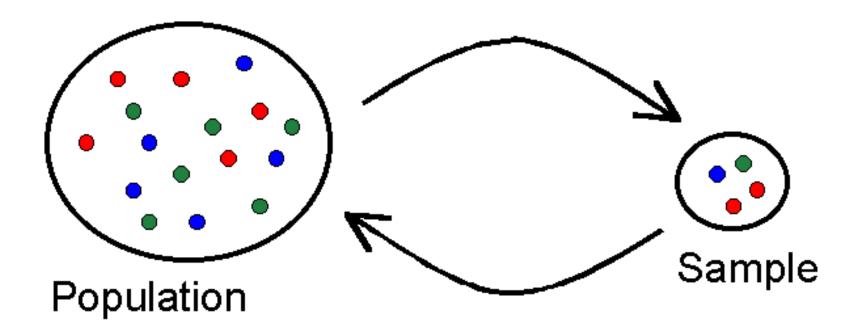
- Sample vs Population
- Variance
- Standard Deviation
- Standard Normal Distribution



## **Sample vs Population**



#### Remember:



#### **Population Variance**



$$\sigma^2 = \sum_{i=1}^n (x_i - \mu)^2$$

Where:

 $\sigma^2$  = variance of the population (pronounced sigma squared)

 $X_i$  = the measurement of each data unit in the population

 $\mu$  = the population mean

n = the size of the population

#### Sample Variance



$$s^{2} = \sum_{i=1}^{n} (x_{i} - \bar{x})^{2}$$

$$n-1$$

Where:

 $S^2$  = the variance of the sample

 $X_i$  = the measurement of each data unit in the sample

 $\overline{\chi}$  = the sample mean

n = the size of the sample (the number of data values)

#### **Standard Deviation - Population**



$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$$

#### Where:

 $\sigma$  = the standard deviation of the population

 $\mathcal{X}_i$  = the measurement of each data unit in the population

 $\mu$  = the population mean

n = the size of the population

#### **Standard Deviation - Sample**



$$S = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}}$$

S = the standard deviation of the sample

 $\mathcal{X}_i$  = the measurement of each data unit in the sample

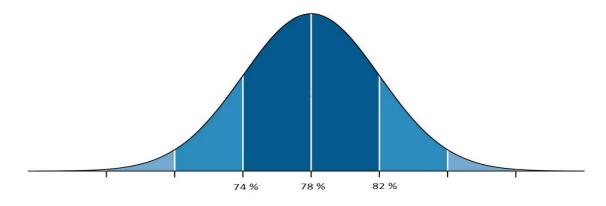
 $\overline{\chi}$  = the sample mean

Where:

n =the size of the sample (the number of data values)



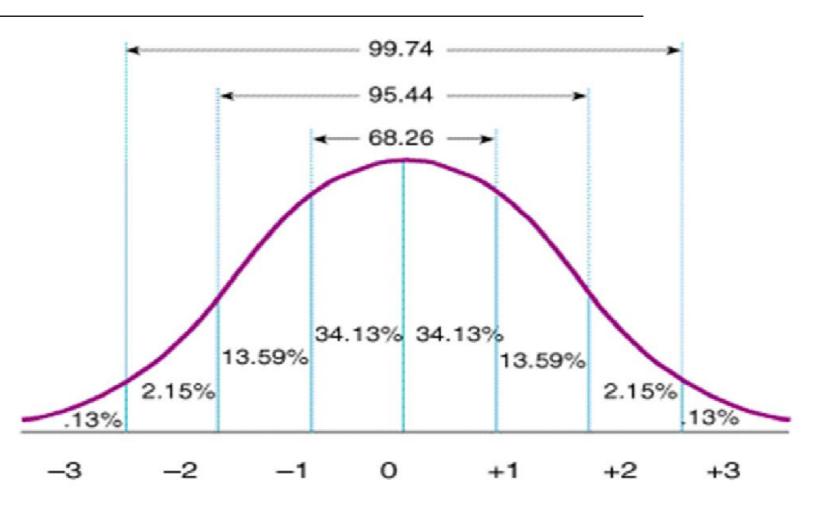
Because the area under the curve = 1 and the curve is symmetrical, we can say the probability of getting more than 78 % is 0.5, as is the probability of getting less than 78 %



To define other probabilities (ie. The probability of getting 81 % or less) we need to define the standard normal distribution



#### **Standard Normal Distribution**



Normal distribution with  $\mu = 0$  and SD = 1



We do this using the following formula

$$z = \frac{x - \mu}{\sigma}$$

 $\chi$  = the normally distributed random variable of interest

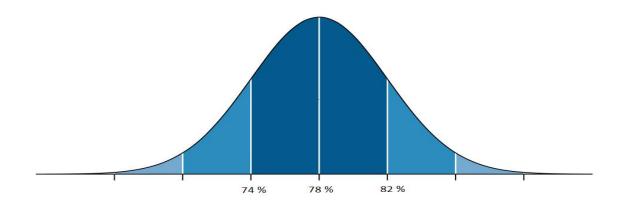
 $\mu$  = the mean for the normal distribution

 $\sigma$  = the standard deviation of the normal distribution

z = the z-score (the number of standard deviations between x and  $\mu$ )



▶ To determine the probability of getting 81 % or less



$$z = \frac{x - \mu}{\sigma} = \frac{81 - 78}{4} = 0.75$$



Now that you have the standard z-score (0.75), use a z-score table to determine the probability

X	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621



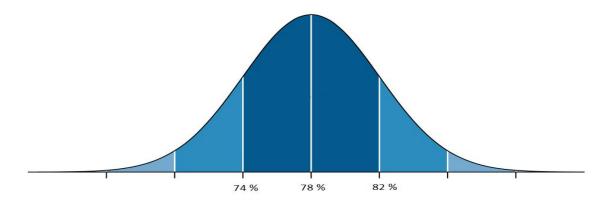
Arr Z = 0.75, in this example, so we go to the 0.7 row and the 0.05 column

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## **Normal Probability Distribution**

- The probability that the z-score will be equal to or less than 0.75 is 0.7734
- Therefore, the probability that the score will be equal to or less than 81 % is 0.7734



▶ There is a 77.34 % chance I will get 81 % or less on my test



#### Sample vs Population Distributions

#### **Learning Intentions**

Today we will understand:

 Using sampling distributions of the mean and proportion



- Working with the central limit theorem
- Using standard error of the mean



#### What is a Sampling Distribution?

The sampling distribution of the mean refers to the pattern of sample means that will occur as samples are drawn from the population at large

#### **Example**

I want to perform a study to determine the number of kilometres the average person in Australia drives a car in one day. It is not possible to measure the number of kilometres driven by every person in the population, so I randomly choose a sample of 10 people and record how far they have driven.



#### What is a Sampling Distribution?

I then randomly sample another 10 drivers in Australia and record the same information. I do this a total of 5 times. The results are displayed in the table below.

Sample Number	Average number of Kilometres
1	25.6
2	50.2
3	15.1
4	43.9
5	36.8



## What is a Sampling Distribution?

- Each sample has its own mean value, and each value is different
- We can continue this experiment by selecting and measuring more samples and observe the pattern of sample means
- This pattern of sample means represents the sampling distribution for the number of kilometres a person the average person in Australia drives



#### Sampling Distribution of the Mean

The distribution from this example represents the sampling distribution of the mean because the mean of each sample was the measurement of interest

What happens to the sampling distribution if we increase the sample size?

Don't confuse sample size (n) and the number of samples. In the previous example, the sample size equals 10 and the number of samples was 5.



#### The Central Limit Theorem

- What happens to the sampling distribution if we increase the sample size?
- As the sample size (n) gets larger, the sample means tend to follow a normal probability distribution
- As the sample size (n) gets larger, the sample means tend to cluster around the true population mean
- Holds true, regardless of the distribution of the population from which the sample was drawn



#### **Standard Error of the Mean**

- As the sample size increases, the distribution of sample means tends to converge closer together – to cluster around the true population mean
- Therefore, as the sample size increase, the standard deviation of the sample means decreases
- The standard error of the mean is the standard deviation of the sample means



#### Standard Error of the Mean

Standard error can be calculated as follows:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Where:

 $\sigma_{ar{X}}$  = the standard deviation of the sample means (standard error)

 $\sigma$  = the standard deviation of the population

 $\sqrt{n}$  = the sample size



#### Standard Error of the Mean

- In many applications, the true value of  $oldsymbol{\sigma}$  (the SD of the population) is unknown
- SE can be estimated using the sample SD

$$SE_{\bar{x}} = \frac{S}{\sqrt{n}}$$

Where:

 $SE_{\bar{x}}$  = the standard deviation of the sample means (standard error)

S = the sample standard deviation (the sample based estimate of the SD of the population

 $\sqrt{n}$  = the sample size



#### **Using the Central Limit Theorem**

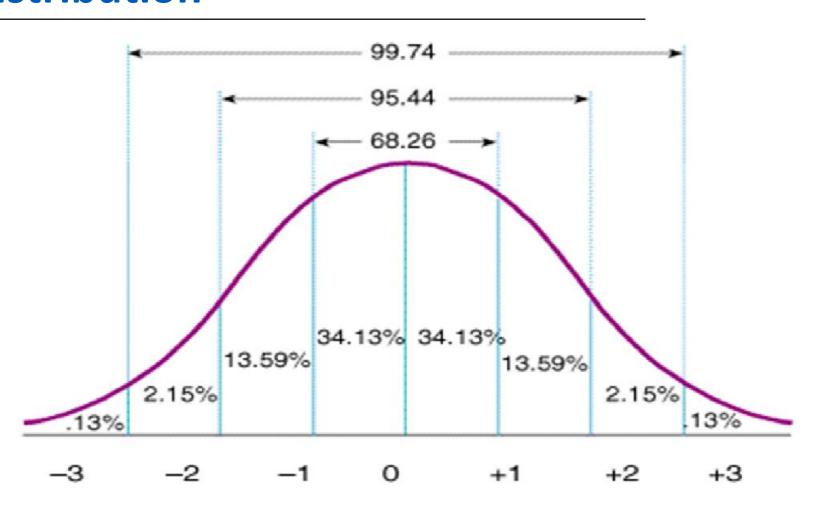
- If we know that the sample means follow the normal probability distribution
- And we can calculate the mean and standard deviation of that distribution

We can predict the likelihood that the sample means will be more or less than certain values

PLAY

# Remember! - Standard Normal Distribution





Normal distribution with  $\mu = 0$  and SD = 1



#### **Using the Central Limit Theorem**

- ▶ As we did last week, we can calculate the z-score
- Determine probability using a standard z table

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