Learning, Teaching and Student Engagement

JAMES COOK UNIVERSITY AUSTRALIA

Mathematics for Physics

The module covers concepts such as:

- **Patterns**
- **Basic Algebra**
- **Solving Equations**
- **Unit Conversion**
- **Algebra Problem Solving**

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Mathematics for Physics

Contents

1. What is Algebra

Algebraic thinking spans all areas of mathematics. It goes beyond simply manipulating symbols, to understanding mathematical structure and exploring generalisations. It involves forming and recognising number relationships, and expressing these relationships using symbols. Developing algebraic thinking is best achieved through practical problem solving.

For example: When describing an odd number, we could say that, 'An odd number is **any number** that when divided by two will leave a remainder of one'. This sentence describes the mathematical concept of odd numbers. We can also describe this relationship using algebra:

"When $\frac{x}{2}$ has a remainder of 1, x is an odd number."

- Importantly "x" can vary and therefore represent any number
- Also note that "x" is not an abbreviation or shorthand, such as "m" for metres, or "p" for perimeter

Hence, algebra provides the written algebra form to firstly express mathematical ideas and then relationships.

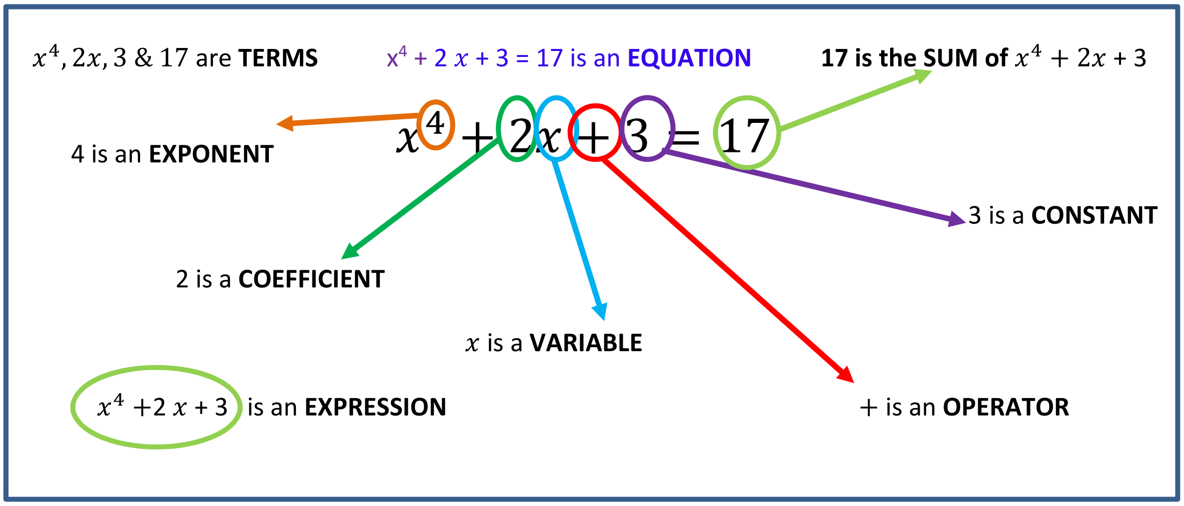
An Example: If I had a bag of apples that were to be shared between four people, I could represent the number of apples with the letter n. The letter 'n' is known as a variable when used in this way. I can express the process of sharing n apples between four people mathematically. Each person will receive $\frac{n}{4}$ apples.

This workbook encourages algebraic thinking and assumes a certain level of mathematical ability. Algebraic thinking is then applied in the form of physics concepts and questions.

If you feel you need more support with other areas of Maths, further resources are available from The Learning Centre: https://www.jcu.edu.au/students/learning-centre/maths-and-statistics

Until the 17^{th} century, algebra was expressed in words without the use of Mathematical symbols.
Describing an equation such as $(10-x)(10+x)=x^2-100$ could be expressed in up to 15 lines!

2. Glossary



Equation: A mathematical sentence containing an equal sign. The equal sign demands that the expressions on either side are balanced and equal.

Expression: An algebraic expression involves numbers, operation signs, brackets/parenthesis and variables that substitute numbers but does not include an equal sign.

Operator: The operation ($+$, $-$,$\times$,$\div$) which separates the terms.

Term: Parts of an expression separated by operators which could be a number, variable or product of numbers and variables. Eg. $2x$, 3 & 17

Variable: A letter which represents an unknown number. Most common is x, but can be any symbol.

Constant: Terms that contain only numbers that always have the same value.

Coefficient: A number that is partnered with a variable. The term $2x$ is a coefficient with variable. Between the coefficient and variable is a multiplication. Coefficients of 1 are not shown.

Exponent: A value or base that is multiplied by itself a certain number of times. Ie. x^4 represents $x \times x \times x \times x$ or the base value x multiplied by itself 4 times. Exponents are also known as Powers or Indices.

In summary:

Variable:	x	Operator:	$+$
Constant:	3	Terms:	3, $2x$ (a term with 2 factors) & 17
Equation:	$2x + 3 = 17$	Left hand expression:	$2x + 3$
Coefficient:	2	Right hand expression	17 (which is the sum of the LHE)

3. Patterns and Simple Algebra

Algebra provides a clear and descriptive way to explain everyday activities using maths. A useful basic application for algebra is to describe and predict patterns. This allows you to create your own algebraic equation describing a pattern that you can draw and confirm.

When representing an unknown number using a variable, the choice of letter is not significant mathematically although a distinctive choice can aid memory. For example, "v" can be used to represent velocity. In the following example "n" represents "Number of Hours" worked.

For example:

The table below represents the salary that Petra earns for various hours of work if she is paid $24 an hour.

Number of Hours	Salary Earned($)
1	24 x 1 = 24
2	24 x 2 = 48
3	24 x 3 = 72
n	24 x n = 24n

In this example, an algebraic expression is used to represent a number pattern. The pattern is described using a general rule or equation: Salary Earned = 24n

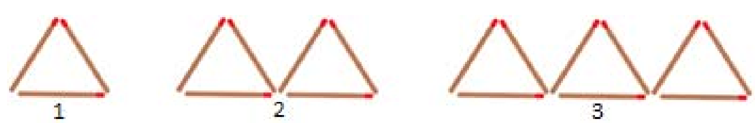
Question 1:

a. Using the provided formula, calculate Petra's earnings after working 12 hours.

b. Describe the formula in sentence form

Question 2:

a. Create a general rule for the pattern below relating "number of triangles" to "number of matches", using algebra where:
n = Number of triangles
m = Number of matches



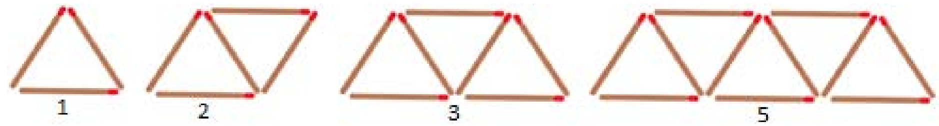
b. Using your formula, calculate the number of matches required for 10 triangles

c. How many triangles could be created with 60 matches?
(If you are uncertain about rearranging the equation, see page 10)

d. Describe the formula in sentence form

Question 3:

a. Complete the table below and create a general rule using algebra to describe the pattern:



Number of triangles (n)	1	2	3	4	5	
Number of matches (m)						

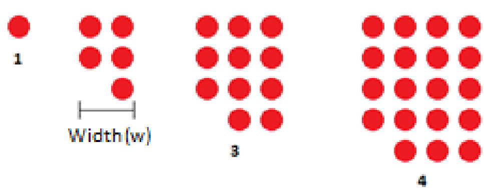
b. Using your formula, calculate the number of matches required for 15 triangles

c. How many triangles could be created with 21 matches?

d. Describe your formula in sentence form

Question 4:

a. Complete the table below and create a general rule (in the form of ax^2+bx+c) to describe the pattern:



Width(w)	1	2	3	4	5	
Number of dots (d)	1	5	11			

b. Using your formula, calculate the number of dots required for a shape that is 10 dots wide

[Extension Question]

c. What length of shape could be created with 419 dots?
https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations

4. Some Basic Algebra Rules

Expressions with zeros and ones:

Zeros and ones can be eliminated. For example:

When zero is added the number isn't changed, $x + 0 = x$ or $x - 0 = x$

$$(6 + 0 = 6, \qquad 6 - 0 = 6)$$

If a number is multiplied by positive 1, the number stays the same, $x \times 1 = x$ or $\frac{x}{1} = x$

$$(6 \times 1 = 6, \ \frac{6}{1} = 6)$$

Note: Using indices (powers), any number raised to the power of zero is 1.

$$\frac{2^2}{2^2} = \frac{4}{4} = 1 \quad or \quad \frac{2^2}{2^2} = 2^{2-2} = 2^0 = 1$$

- Additive Inverse: $x + (-x) = 0$
- Any number multiplied by its reciprocal equals one. $x \times \frac{1}{x} = 1;\ 4 \times \frac{1}{4} = 1$
- Symmetric Property: $If\ x = y\ then\ y = x$
- Transitive Property: $If\ x = y\ and\ y = z, then\ x = z$

  For example, if apples cost $2 and oranges cost $2 then apples and oranges are the same price.

The **Order of Operations** is remembered using the mnemonic known as the BIDMAS or BOMDAS (Brackets, Order, Multiplication/Division, and Addition/Subtraction).

Also a **golden rule**:

"What we do to one side we do to the other"

Question 5:

Here are some revision examples for practise:

a. $10 - 2 \times 5 + 1 =$

b. $10 \times 5 \div 2 - 3 =$

c. $12 \times 2 - 2 \times 7 =$

d. $48 \div 6 \times 2 - 4 =$

e. $What\ is\ the\ missing\ operation\ symbol\ 18\ \_\ 3 \times 2 + 2 = 14$

5. Addition & Multiplication Properties

Maths Property	**Rule**	**Example**
Commutative The number order for addition or multiplication doesn't affect the sum or product	$a + b = b + a$ $ab = ba$	$1 + 3 = 3 + 1$ $2 \times 4 = 4 \times 2$
Associative Since the Number order doesn't matter, it may be possible to regroup numbers to simplify the calculation	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$	$1 + (2 + 3) = (1 + 2) + 3$ $2 \times (2 \times 3) = (2 \times 2) \times 3$
Distributive	$a(b + c) = ab + ac$	$2(3 + 1) = 2 \times 3 + 2 \times 1$
Zero Factor	$a \times 0 = 0$ If $ab = 0$, then either $a = 0$ or $b = 0$	$2 \times 0 = 0$
Rules for Negatives	$-(-a) = a$ $(-a)(-b) = ab$ $-ab = (-a)b = a(-b) = -(ab)$ $(-1)a = -a$	$-(-3) = 3$ $(-2)(-3) = 2 \times 3$ $-2 \times 3 = (-2) \times 3$ $= 2 \times (-3)$ $= -(2 \times 3)$ $(-1) \times 2 = -2$
Rules for Division	$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ $\frac{-a}{-b} = \frac{a}{b}$ $If\ \frac{a}{b} = \frac{c}{d}\ then\ ad = bc$ $Proof$: $\frac{a}{b} = \frac{c}{d}$ $\frac{\cancel{b} \times a}{\cancel{b}} = \frac{bc}{d}$ (multiply everything by b) $a = \frac{bc}{d}$ $a \times d = \frac{bc \times \cancel{d}}{\cancel{d}}$ (multiply by d) $ad = bc$	$-\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2}$ $\frac{-6}{-3} = \frac{6}{3}$ $If\ \frac{1}{2} = \frac{3}{4}\ then\ 1 \times 4$ $= 2 \times 3$

6. Collecting Like Terms

Algebraic thinking involves simplifying problems to make them easier to solve. The problem below consists of several related or "like" terms. Like terms can be grouped or collected, creating a smaller or simpler question.

$$7x + 2x + 3x - 6x + 2 = 14$$

A **like term** is a term which has the **same variable** (it may also have the same power/exponent/index), with only a **different coefficient**. In the equation above, there are four different coefficients $(7, 2, 3, \& \ 6)$ with the same variable, x, and no exponents to consider.

Like terms (multiplied by 'x') can be collected: $(\mathbf{7x + 2x + 3x - 6x}) + 2 = 14$
(+2 isn't a like term as it doesn't share the variable 'x')

The coefficients can be added and subtracted separate to the variables: $7 + 2 + 3 - 6 = 6$
Therefore: $7x + 2x + 3x - 6x = 6x$

The original equation simplifies to $6x + 2 = 14$
Now we solve the equation:

$$6x + 2(-2) = 14 - 2$$
$$6x = 12$$
$$6\,x(\div 6) = 12 \ \div 6$$
$$x = 2$$

EXAMPLE PROBLEM:

1. Collect the like terms and simplify:

$$5x + 3xy + 2y - 2yx + 3y^2$$

Step 1: Recognise the like terms (note: xy is the same as yx; commutative property)

$$5x + 3xy + 2y - 2yx + 3y^2$$

Step 2: Arrange the expression so that the like terms are together (remember to take the operator with the term).

$$5x + 2y + 3xy - 2yx + 3y^2$$

Step 3: Simplify the equation by collecting like terms: $5x + 2y + 1xy + 3y^2$
Note: a coefficient of 1 is not usually shown $\therefore$ $5x + 2y + xy + 3y^2$

Question 6

Simplify:

a. $3x + 2y - x$

b. $3m + 2n + 3n - m - 7$

c. $2x^2 - 3x^3 - x^2 + 2x$

Expand the brackets then collect like terms

d. $3(m + 2n) + 4(2m + n)$

e. $4(x + 7) + 3(2x - 2)$

7. Solving Equations

The equal sign of an **equation** indicates that both sides of the equation are equal. The equation may contain an **unknown quantity** (or variable) whose value can be calculated. In the equation, $5x + 10 = 20$, the unknown quantity is x. This means that 5 multiplied by something (x) and added to 10, will equal 20.

- To solve an equation means to find all values of the unknown quantity so that they can be substituted to make the left side and right sides **equal**
- Each such value is called a **solution** (e.g. $x = 2$ in the equation above)
- The equation is rearranged and solved in reverse order of operation (SADMOB – see page 7) to find a value for the unknown

  Eg. $5x + 10 = 20$ **First rearrange by subtracting 10 from the LHS and the RHS**

  $5x + 10(-10) = 20(-10)$

  $5x = 10$

  $\frac{5x}{5} = \frac{10}{5}$ **Divide both LHS and RHS by 5**

  $x = 2$

  To check the solution, substitute x *for* 2 ; $(5 \times 2) + 10 = 20 \therefore 10 + 10 = 20$

Four principles to apply when solving an equation:

1. **Work towards solving the variable:** $(x =)$
2. **Use the opposite mathematical operation**: Remove a constant or coefficient by doing the opposite operation on both sides:

   Opposite of $\times$ *is* $\div$

   Opposite of $+$ *is* $-$

   Opposite of x^2 *is* $\sqrt{x}$

   Opposite of $\sqrt{x}$ *is* $\pm x^2$
3. **Maintain balance**: "What we do to one side, we must do to the other side of the equation."
4. **Check:** Substitute the value back into the equation to see if the solution is correct.

One-step Equations

Addition	Subtraction
$x + (-5) = 8$ $x + (-5) - (-5) = 8 - (-5)$ $x = 8 + 5$ $\therefore x = 13$	$x - 6 + 6 = (-4) + 6$ So $x = (-4) + 6$ $\therefore x = 2$
Check by substituting 13 for x $13 + (-5) = 8$ $13 - 5 = 8$ $8 = 8$ ✓	Check by substituting 2 for x $2 - 6 = (-4)$ ✓ $-4 = -4$

Question 7

Solve for x:

a. $x + 6 - 3 = 18$

b. $7 = x + (-9)$

c. $x - 12 = (-3)$ d. $18 - x = 10 + (-6)$

Two-step Equations

The following equations require two steps to single out the variable (i.e. To solve for x).

ADDITION EXAMPLE: $2x + 6 = 14$

Step 1: The constant 6 is subtracted from both sides, creating the following equation:

$$2x + 6 - 6 = 14 - 6$$ **(The opposite of $+6$ is -6)**

$$2x = 8$$

Step 2: Next both sides are divided by two, creating the following equation:

$$\frac{2x}{2} = \frac{8}{2}$$ **(The opposite of $2x$ is $\div$ 2)**

$$\therefore x = 4$$

Check. Substitute x = 4 into the equation: $2 \times 4 + 6 = 14$

$$8 + 6 = 14$$

$$14 = 14$$ ✓ **(The answer must be correct)**

SUBTRACTION EXAMPLE:

Solve for *j*: $3j - 5 = 16$

Step 1: $3j - 5 = 16$ (deal with the subtraction first)

$3j - 5 + 5 = 16 + 5$ (Opposite of -5 is $+5$)

Thus, $3j = 21$ (the multiply second)

Step 2: $\frac{3j}{3} = \frac{21}{3}$ (the opposite of $\times 3$ is $\div 3$)

$\therefore j = 7$

Check: $3 \times 7 - 5 = 16$ ✓

MULTI-STEP EXAMPLE:

Solve for *T*: $\frac{3T}{12} - 7 = 6$

$$\frac{3T}{12} - 7 = 6$$

$$\frac{3T}{12} - 7 + 7 = 6 + 7$$

$$\frac{3T}{12} = 13$$

$$\frac{3T}{12} \times 12 = 13 \times 12$$

$$3T = 156$$

$$\frac{3T}{3} = \frac{156}{3}$$

$$\therefore T = 52$$

Check: $(3 \times 52) \div 12 - 7 = 6$ ✓

Question 8:

Solve the following to calculate the unknown variable:

a. $5x + 9 = 44$

b. $\frac{x}{9} + 12 = 30$

c. $3y + 13 = 49$

d. $4x - 10 = 42$

e. $\frac{x}{11} + 16 = 30$

8. Rearranging Formulas

A formula uses symbols and rules to describe a relationship between quantities. In mathematics, formulas follow the standard rules for algebra and can be rearranged as such. If values are given for all other variables described in the formula, rearranging allows for the calculation of the unknown variable. This is a mathematical application of working with variables and unknowns in physics and engineering problems. In this section, rearranging equations and substituting values is practiced with common physics formulas to determine unknowns. The next section applies these skills to problem solving including application of appropriate units.

Basic Rearranging Example:

Calculate the density $(\boldsymbol{\rho})$ of Lithium

Given: Mass (m) = 268 Volume (v) = 0.5

$\boldsymbol{\rho} = \frac{m}{V}$ [density is defined by this relationship]

$$\boldsymbol{\rho} = \frac{268}{0.5}$$

$$\boldsymbol{\rho} = 536$$

Alternatively, calculate the Mass (m) of Lithium

Given: Density (ρ) = 536 and Volume (V) = 0.5

$\rho = \frac{\boldsymbol{m}}{V}$

$V \times \rho = \frac{\boldsymbol{m} \times \cancel{V}}{\cancel{V}}$ (rearrange to calculate m)

$$V\rho = \boldsymbol{m}$$

$\boldsymbol{m} = 0.5 \times 536$ (substitute values and calculate)

$$\boldsymbol{m} = 268$$

Finally, calculate the Volume (V) of Lithium

Given: Density (ρ) = 536 and Mass (m) = 268

$\rho = \frac{m}{\boldsymbol{V}}$

$\boldsymbol{V}\rho = m$ (as above)

$$\frac{\boldsymbol{V}\cancel{\rho}}{\cancel{\rho}} = \frac{m}{\rho}$$

$$\boldsymbol{V} = \frac{m}{\rho}$$

$\boldsymbol{V} = \frac{268}{536}$ = 0.5

Many other equations use exactly the same process of rearranging

For example: $\vec{a} = \frac{\Delta\vec{v}}{t}$ $\vec{F} = m\vec{a}$ $P = \frac{F}{A}$ $v = \frac{d}{t}$ and $M = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$

Note that each formula describes a relationship between values:
$\vec{a} = acceleration\ vector$
$\vec{v} = velocity\ vector$
$t = time$
$\vec{F} = force$
$m = mass$
$P = pressure$
$A = area$
$d = displacement$
$M = magnification$
$d_i = image\ distance$
$d_o = object\ distance$
$h_i = image\ height$
$h_o = object\ height$

Question 9:

a. Using $v = f\lambda$
   i. Given f = 3 and λ = 9, calculate v

   ii. Given v = 40 and λ = 5, rearrange the equation to calculate f

   iii. Given v = 60 and f = 3, rearrange the equation to calculate λ

b. Using $\vec{F} = m\vec{a}$
   i. Given m = 12 and $\vec{a}$ = 4, calculate $\vec{F}$

   ii. Given $\vec{F}$= 81 and $\vec{a}$ =3, calculate m

   iii. Given $\vec{F}$= 48 and calculate m=12, calculate $\vec{a}$

c. Using $\vec{F}\Delta t = \Delta\vec{p}$
   i. Given Δt = 5 and $\Delta\vec{p}$ = 50, rearrange the equation to calculate $\vec{F}$

   ii. Given $\vec{F}$ = 5 and $\Delta\vec{p}$ = 30, rearrange the equation to calculate Δt

d. Using $\vec{a} = \frac{\Delta\vec{v}}{t}$

i. Given $\Delta\vec{v}$ = 60 and t = 12, calculate $\vec{a}$

ii. Given $\Delta\vec{v}$ = 124 and $\vec{a}$ = 4 , calculate t

e. Using $F_f = \mu F_N$

i. Given F_f = 4416 and μ = 48, calculate F_N

ii. Given F_f = 1050 and F_N = 42, calculate μ

f. Using $\vec{x} = \frac{\overrightarrow{v_1}+\overrightarrow{v_2}}{2} \times t$

i. Given $\overrightarrow{v_1}$ = 15 , $\overrightarrow{v_2}$ = 25 and t = 10, calculate $\vec{x}$

ii. Given $\overrightarrow{v_1}$ = 60 , $\overrightarrow{v_2}$ = 90 and $\vec{x}$ = 225, calculate t

g. Using $v_2{}^2 = v_1{}^2 + 2ax$

   i. Given v_1 = 9 , a = 6 and x = 12, calculate v_2

   ii. Given v_2 = 10 , a = 8, x = 4, calculate v_1

   iii. Given v_2 = 12 , a = 8, v_1 = 8, calculate x

h. Using $E_k = \frac{mv^2}{\mathbf{2}}$

   i. Given E_k = 96 and v=4, calculate m

9. Unit Conversion

Measurements consist of two parts – the number and the identifying unit.

Number → **50 kg** ← Identifying unit

In scientific measurements, units derived from the metric system are the preferred units. The metric system is a decimal system in which larger and smaller units are related by factors of 10.

Table 1: Common Prefixes of the Metric System

Prefix	Abbreviation	Relationship to Unit	Exponential Relationship to Unit	Example
mega-	M	1 000 000 x Unit	10^{6} x Unit	2.4ML -Olympic sized swimming pool
kilo-	k	1000 x Unit	10^{3} x Unit	The average newborn baby weighs 3.5kg
-	-	Units	Unit	metre, gram, litre, sec
deci-	d	1/10 x Unit **or** 0.1 x Unit	10^{-1} x Unit	2dm - roughly the length of a pencil
centi-	c	1/100 x Unit **or** 0.01 x Unit	10^{-2} x Unit	A fingernail is about 1cm wide
milli-	m	1/1000 x Unit **or** 0.001 x Unit	10^{-3} x Unit	A paperclip is about 1mm thick
micro-	µ	1/1 000 000 x Unit **or** 0.000001 x Unit	10^{-6} x Unit	human hair can be up to 181 µm
nano-	n	1/1 000 000 000 x Unit **or** 0.000000001 x Unit	10^{-9} x Unit	DNA is 5nm wide

Table 2: Common Metric Conversions

Unit	Larger Unit	Smaller Unit
1 metre	1 kilometre = 1000 metres	100 centimetres = 1 metre 1000 millimetres = 1 metre
1 gram	1 kilogram = 1000 grams	1000 milligrams = 1 gram 1 000 000 micrograms = 1 gram
1 litre	1 kilolitre = 1000 litres	1000 millilitres = 1 litre

Example:

Convert 0.15 g to kilograms and milligrams	Convert 5234 mL to litres
Because 1 kg = 1000 g, 0.15 g can be converted to kilograms as shown: $$\mathbf{0.15\ g\ x\ \frac{1\ kg}{1000\ g} = 0.00015\ kg}$$ Also, because 1 g = 1000 mg, 0.15 g can be converted to milligrams as shown: $$\mathbf{0.15\ \cancel{g}\ x\ \frac{1000\ mg}{1\ \cancel{g}} = 150\ mg}$$	Because 1 L = 1000 mL, 5234 mL can be converted to litres as shown: $$\mathbf{5234\,mL\ x\ \frac{1\ L}{1000\ mL} = 5.234\ L}$$

Question 10:

1 a) Convert 600 g to kilograms and milligrams

b) Convert 4.264 L to kilolitres and millilitres

c) Convert 670 cm to metres and kilometres

10. Working with Units

In real world mathematical applications, physical quantities include units. Units of measure (eg. metres, Litres or kilograms) describe quantities based on observations or measurement. Units can also describe mathematical relationships between measurements of different properties. Different quantities of the same unit can be directly compared numerically, e.g. 4kg of sugar is much greater than 1kg of sugar. However it is meaningless to compare measurements of different units, e.g. 4m and 40min are not comparable. It is important to understand correct working with units if they are to be used effectively.

Table 3: Base SI Units and Associated Symbol:

Base Unit Name	Unit Symbol	Quantity
Metre	m	Length
Kilogram	kg	Mass
Second	s	Time
Ampere	A	Electric Current
Kelvin	K	Temperature
Mole	mol	Amount of a Substance
Candela	cd	Luminous Intensity

Two Methods:

Method 1: Base Unit Method

The International System of Units (SI) describes physical properties in what are known as base units. The base unit of length, mass, time are metres, kilograms and seconds respectively (see Table 3). Base units are derived from constants of nature such as the speed of light in a vacuum ($\text{e.g. } 1m = \text{distance travelled by light in a vacuum in } \frac{1}{299\,792\,458} \text{ seconds}$) which can be measured with great accuracy. A unit that is derived from another unit is not considered a base unit (e.g. Volume in unit m^3 is derived from metres). Most formulas are written using base units, meaning any value in the correct base unit form can be used.

Base Unit Example:

The specific heat of gold is $129 \frac{J}{kg \cdot K}$*. What is the quantity of heat energy required to raise the temperature of 100 g of gold by 50.0K?*

The solution is derived from the specific heat formula: $\mathrm{Q} = \mathrm{mc}\Delta\mathrm{T}$

Q=heat energy (base unit Joules, J)
m = mass (base unit kilograms, kg)

c = specific heat (derived unit $\frac{J}{kg \cdot K}$)
ΔT = change in temperature (T) (base unit Kelvin, K)

Given:

The mass is currently in the mass unit g and must first be converted to the SI base unit kg.

$$m = 100g \times \frac{1kg}{1000g} = 0.1kg$$

$$c = 129 \frac{J}{kg \cdot K}$$

$$\Delta T = 50K$$

The unit for specific heat & temperature are already in their base SI unit and do not require conversion.

Required:

$$Q = Heat\ Energy\ in\ J$$

Analysis:

Applying the values to the variables in the equation gives:

$$Q = 0.1 \times 129 \times 50$$

Solution:

$Q = 645J$ (Joules is the base SI unit for heat energy)

∴ The quantity of heat required to raise the temperature of 100g of gold to 50K is 645J

Method 2: Factor Label Method

Expressing a formula with units and working with units algebraically (or "dimensional analysis") has several advantages, especially when working with non-standard units. Units can be converted, the change from initial units to final unit is immediately clear and the formula may even be derived directly from the final unit. The process of working with algebra associates variables with units, allowing for the substitution and cancellation of units.

Unit Conversion Example:

Convert 10km per hour into metres per second.

This question can be split into 3 parts: find a conversion factor for kilometres to metres, find a conversion factor for hours to seconds, and multiply the initial value by the conversion factors to find a final value in correct units.

$$\boldsymbol{Initial\ Value \times Conversion\ Factors = Final\ Value}$$

1. The relationship between kilometres and metres is 1km = 1000m. This is used to create a conversion factor.
    a. 10km/hr is written as a unit fraction (10km is travelled per 1 hr)
    $$\frac{10km}{1hr}$$
    b. The unit being converted is multiplied on the opposite side of the fraction
    $$\frac{10km}{1hr} \times \frac{}{km}$$
    Converting from km which was on top so written on bottom
    c. The desired final unit is then written to complete the fraction
    $$\frac{10km}{1hr} \times \frac{m}{km}$$
    Converting to m
    d. The new fraction is made to equal 1 using the relationship between units. The conversion factor is now created.
    $$\frac{10km}{1hr} \times \frac{1000m}{1km}$$
    Remember: $\frac{1km}{1000m}$ or $\frac{1000m}{1km} = 1$

2. The relationship between hours and minutes is 1hour = 60min, and the relationship between minutes and seconds is 1min = 60sec. This conversion will take 2 steps and create 2 conversion factors.
    a. 10km/hr is written as a unit fraction
    $$\frac{10km}{1hr}$$
    b. The unit being converted is multiplied on the opposite side of the fraction
    $$\frac{10km}{1hr} \times \frac{hr}{}$$
    c. The desired final unit is written to complete the fraction
    $$\frac{10km}{1hr} \times \frac{hr}{min}$$
    d. The new fraction is made to equal 1 using the relationship between units. The conversion factor is now created.
    $$\frac{10km}{1hr} \times \frac{1hr}{60min}$$
    e. The process is repeated to convert minutes to seconds
    $$\frac{10km}{1hr} \times \frac{1hr}{60min} \times \frac{1min}{60sec}$$

3. The initial value of 10km/hr is then multiplied by the conversion factors:
$$\frac{10km}{1hr} \times \frac{1000m}{1km} \times \frac{1hr}{60min} \times \frac{1min}{60sec}$$

4. Values and units appearing on both the top and bottom of the division cancel out. (Any value divided by itself is equal to 1 (ie. $2 \div 2 = 1$ or $km \div km = 1$) and anything multiplied by 1 is itself)

$$\frac{10\cancel{km}}{\cancel{1hr}} \times \frac{1000m}{\cancel{1km}} \times \frac{\cancel{1hr}}{60\cancel{min}} \times \frac{\cancel{1min}}{60sec}$$

5. The remaining values and units are multiplied and divided following normal conventions.
$$\frac{10 \times 1000m}{60 \times 60sec} = \frac{10000m}{3600sec} = \frac{100m}{36sec} = 2.78m/sec = 2.78\text{m/s}$$

Solving the Equation Example:

The specific heat of gold is 129 J/kg·K. What is the quantity of heat energy required to raise the temperature of 100 g of gold by 50.0K?

The formula for specific heat is: $Q = mC\Delta T$

Variables:
Q = units J
m = 100g
C = 129 J/kg·K
ΔT = 50K

$$Q = mC\Delta T$$

$$Q = 100g \times \frac{129J}{kg \times K} \times 50K$$ **Units g and kg are both units of mass but incompatible with each other. One must be converted to the other.**

$$Q = \frac{100g}{1} \times \frac{1kg}{1000g} \times \frac{129J}{kg \times K} \times \frac{50K}{1}$$ **A conversion factor is needed (1kg = 1000g)**

$$Q = \frac{100\cancel{g}}{1} \times \frac{1\cancel{kg}}{1000\cancel{g}} \times \frac{129J}{\cancel{kg} \times \cancel{K}} \times \frac{50\cancel{K}}{1}$$ **Units are cancelled where they appear either side of the fraction**

$$Q = \frac{100 \times 1 \times 129 \times 50J}{1 \times 1000 \times 1}$$ **Equation is solved using normal conventions**

$$Q = \frac{645000J}{1000} = 645J$$ **Final and remaining unit is Joules (J)**

∴ The quantity of heat required to raise the temperature of 100g of gold to 50K is 645J

Question 11:

a. A baby elephant walks at a constant velocity of 0.5 m/sec, having a kinetic energy of 14.125J. What is the mass of the baby elephant?

The formula for specific heat is: $E_k = \frac{mv^2}{2}$

Variables:
E_k = Kinetic Energy = 320, 000J
v = velocity = 0.5m/s
m = mass in units kg

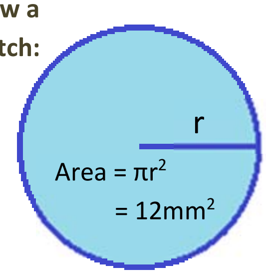
b. Calculate the mass of the baby elephant if it walks at a constant velocity of 2km/h?

11. An Approach to Solving a Problem

Problems involving mathematical relationships between two or more variables can be expressed using algebraic formula. The algebraic expression of these relationships allows for mathematically sound manipulation of variables to investigate, list and calculate different aspects of their relationship.

1. **Put the problem in your own words and/or draw a sketch**
2. **Pull out the facts given in the question**
3. **What do you already know?**
4. **What are the units? Do you need to complete conversions?**
5. **What will the answer look like (and in what units)?**

Draw a sketch:



For Example:

A circle has an area of $12mm^2$, what is its radius?

Put it in your own words: The question asks for the radius of a circle, given an area of **$12mm^2$**.

What do I already know?:
Area: A = $12mm^2$

The formula for area of a circle is Area = πr^2

What are the units? Is a conversion needed? What will the answer look like?:
Calculate radius (r) using area in mm2. The radius will therefore be in mm.

Solve:

$$A = \pi r^2$$

$$12mm^2 = \pi r^2$$ Substitute in known variables

$$\frac{12mm^2}{\pi} = r^2$$

$$r = \sqrt{\frac{12mm^2}{\pi}}$$ Rearrange to solve for **r**

$$r = \pm 1.954mm$$ Solve the equation

Radius is a length that must be positive. A negative radius can be rejected. The radius of a circle with an area of $12mm^2$ is 1.95mm.

It is important to always have a problem solving process that is simple and familiar when answering questions in a test. The process may not be used on every question and will be refined over time and regular use. When stuck and all else has failed, you'll be glad that you at least have a plan.

Geometry

Question 12:

a. A circle has an area of 25cm, what is its radius? ($A = \pi r^2$)

b. A circle has a circumference of 38cm, what is its radius? ($C = 2\pi r$)

c. A cube has a volume of $125mm^3$, what is its surface area?
($volume\ of\ cube = length^3$ & $area\ of\ one\ side = length^2$)

Forces and Motion

Question 13:

a. An airplane flies at 600km/h. It flew for 240min. How far did it travel in kilometres? ($v = \frac{d}{t}$)

b. A force of 250N is applied to an object that accelerates at a rate $5m/sec^2$. What is the mass of the object? (F=ma)

c. A dropped object near the earth will accelerate (a) downward at $9.8\ ms^{-2}$. If the initial velocity (v_0) is 1 m/s downward, what will be its velocity (v) at the end of t=3 sec?
($v = at + v_0$)

Challenge Questions:

Question 14:

a. The security tower for a palace is on a small square piece of land 20m by 20m with a moat of width xm the whole way around.

   a. Find an expression, in expanded form, for the entire area occupied by the moat and the land

   b. Write an expression for the area of the moat.

Question 15:

A cheetah can accelerate from rest to a speed of 126km/h in 7.00 s. What is its acceleration (in ms^{-2})?
($v = at + v_0$)

Question 16:

Mercury has a density of $13.56 g/cm^3$. Calculate the weight of 2L of mercury.
($\rho = \frac{m}{v}$, where $\rho = density \left(\frac{g}{cm^3}\right)$)

12. Answers

1. a. 288 **b.** Petra earns $24 for every hour worked

2. a. m = 3n **b.** 30 matches **c.** 20 triangles

3. a. m = 2n + 1 **b.** 31 c. 10

4. a. $d = w^2 + w - 1$ **b.** 109 **c.** 20 (see note)

5. a. 1 **b.** 22 **c.** 10 **d.** 12 **e.** ÷

6. a. 2x + 2y b. 2m + 5n -7 c. $-3x^3 - x^2 - 2x$ d. 11m + 10n e. 10x + 22

7. a. x = 15 **b.** x = 16 **c.** x = 9 **d.** x = 14

8. a. x = 7 **b.** x = 162 **c.** y = 12 **d.** x = 13 **e.** x = 154

9. a. i. 27 ii. 8 iii. 20 **b.** i. 48 ii. 27 iii. 4 **c.** i. 10 ii. 6 **d.** i. 5 ii. 31 **e.** i. 92 ii. 25

f. i. 200 ii. 3 **g.** i. 15 ii. 6 iii. 5 **h.** i. 12 ii.12

10. a. 0.6kg, 600,000mg b. 0.004264kL, 4264mL c. 6.7m, 0.0067km

11. a. 113kg b. 91.4kg

12. a. 2.82cm **b.** 6.05cm **c.** $150mm^2$

13. a. 2400km **b.** 50kg **c.** 30.4m/s

14. a. $4x^2 + 80$ **b.** $4x^2 + 80x + 400$ **15.** $5ms^{-2}$ **16.** m = 27120g or 17.12kg

Q4. Working:
$419 = w^2 + w - 1$
$0 = w^2 + w - 420$ (a quadratic)
Factorize:
$0 = (w + 21)(w - 20)$
w=-21 or w=+20
You can't have a negative width, therefore the answer is 20

13. Helpful Websites

Algebraic Expressions: https://www.khanacademy.org/

BIDMAS: https://www.educationquizzes.com/

Commutative, Associative and Distributive Laws: https://www.mathsisfun.com/

Learning Algebra: Pre-algebra: https://www.lynda.com/

Like Terms: https://www.freemathhelp.com/

Rearranging Equations: https://www.khanacademy.org/

Solving Equations: https://www.khanacademy.org/

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