Power Operations & Scientific Notation

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1. Power Operations

Powers are also called **exponents** or **indices**; we can work with the **indices** to simplify expressions and to solve problems.

Some key ideas:

a) Any base number raised to the power of 1 is the base itself: for example, $5^1 = 5$
b) Any base number raised to the power of 0 equals 1, so: $4^0 = 1$
c) Powers can be simplified if they are **multiplied** or **divided** and have the **same** base.
d) Powers of powers are multiplied. Hence, $(2^3)^2 = 2^3 \times 2^3 = 2^6$
e) A negative power indicates a reciprocal: $3^{-2} = \frac{1}{3^2}$

Certain rules apply and are often referred to as: **Index Laws.**

Below is a summary of the index rules:

Index Law	Substitute variables for values
$\boldsymbol{a^m \times a^n = a^{m+n}}$	$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
$\boldsymbol{a^m \div a^n = a^{m-n}}$	$3^6 \div 3^3 = 3^{6-3} = 3^3 = 27$
$\boldsymbol{(a^m)^n = a^{mn}}$	$(4^2)^5 = 4^{2\times5} = 4^{10} = 1048\,576$
$\boldsymbol{(ab)^m = a^m b^m}$	$(2 \times 5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100$
$\boldsymbol{({}^a/_b)^m = a^m \div b^m}$	$(10 \div 5)^3 = 2^3 = 8;\ (10^3 \div 5^3) = 1000 \div 125 = 8$
$\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$\boldsymbol{a^{\frac{1}{m}} = \sqrt[m]{a}}$	$8^{1/3} = \sqrt[3]{8} = 2$
$\boldsymbol{a^0 = 1}$	$6^3 \div 6^3 = 6^{3-3} = 6^0 = 1; (6 \div 6 = 1)$

EXAMPLE PROBLEMS:

a) Simplify $6^5 \times 6^3 \div 6^2 \times 7^2 + 6^4 =$

$$= 6^{5+3-2} \times 7^2 + 6^4$$

$$= 6^6 \times 7^2 + 6^4$$

b) Simplify $g^5 \times h^4 \times g^{-1} =$

$$= g^5 \times g^{-1} \times h^4$$

$$= g^4 \times h^4$$

KHANACADEMY

Watch this short Khan Academy video for further explanation:
"Simplifying expressions with exponents"
https://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/simplifying-expressions-with-exponents

Question 1:

a) *Apply the index laws/rules:*

i. Simplify $5^2 \times 5^4 + 5^2 =$

ii. Simplify $x^2 \times x^5 =$

iii. Simplify $4^2 \times \mathrm{t}^3 \div 4^2 =$

iv. Simplify $(5^4)^3 =$

v. Simplify $\frac{2^4 3^6}{3^4} =$

vi. Simplify $3^2 \times 3^{-5} =$

vii. Simplify $\frac{9(x^2)^3}{3xy^2} =$

viii. Simplify $a^{-1}\sqrt{a} =$

b) What is the value of x for the following?

i. $49 = 7^x$

ii. $\frac{1}{4} = 2^x$

iii. $88 = 11^1 \times 2^x$

iv. $480 = 2^x \times 3^1 \times 5^1$

v. Show that $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$

2. Scientific Notation

Numbers as multiples or fractions of ten	Number	Number as a power of ten
10 x 10 x 10	1000	10^3
10 x 10	100	10^2
10	10	10^1
10 x 1/10	1	10^0
1/10	0.1	10^{-1}
1/100	0.01	10^{-2}
1/1000	0.001	10^{-3}

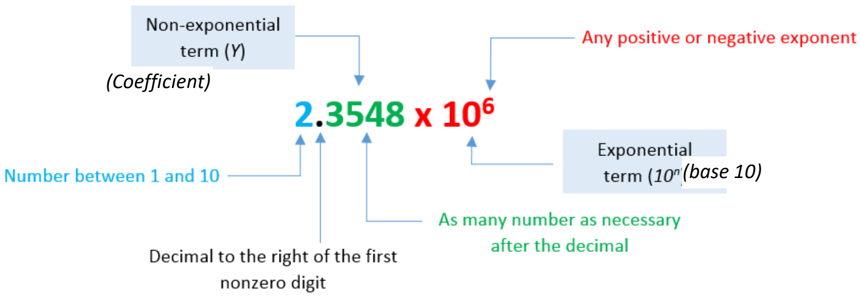
Scientific notation is a convenient method of representing and working with very large and very small numbers. Transcribing a number such as 0.000000000000082 or 5480000000000 can be frustrating since there will be a constant need to count the number of zeroes each time the number is used. Scientific notation provides a way of writing such numbers easily and accurately.

Scientific notation requires that a number is presented as a non-zero digit followed by a decimal point and then a power (exponential) of base 10. The exponential is determined by counting the number places the decimal point is moved.

The number 65400000000 in scientific notation becomes 6.54×10^{10}.

The number 0.00000086 in scientific notation becomes 8.6×10^{-7}.

(Note: $10^{-6} = \frac{1}{10^6}$.)



If *n* is positive, shift the decimal point that many places to the right.

If *n* is negative, shift the decimal point that many places to the left.

Question 2:

Write the following in scientific notation:

a. 450

b. 90000000

c. 3.5

d. 0.0975

Write the following numbers out in full:

e. 3.75×10^2

f. 3.97×10^1

g. 1.875×10^{-1}

h. -8.75×10^{-3}

3. Calculations with Scientific Notation

Multiplication and *division* calculations of quantities expressed in scientific notation follow the index laws since they all they all have the common base, i.e. base 10.

Here are the steps:

Multiplication	Division
A. **Multiply** the coefficients	1. **Divide** the coefficients
B. **Add** their exponents	2. **Subtract** their exponents
C. Convert the answer to scientific Notation	3. Convert the answer to scientific Notation
Example: $(7.1 \times 10^{-4}) \times (8.5 \times 10^{-5})$ $7.1 \times 8.5 = 60.35$ (multiply coefficients) $10^{-4} \times 10^{-5} = 10^{(-4+-5)=-9}$ (add exponents) $= 60.35 \times 10^{-9}$ – check it's in scientific notation ✗ $= 6.035 \times 10^{-8}$ – convert to scientific notation ✓	**Example:** $(9 \times 10^{20}) \div (3 \times 10^{11})$ $9 \div 3 = 3$ (divide coefficients) $10^{20} \div 10^{11} = 10^{(20-11)=9}$ (subtract exponents) $= 3 \times 10^{9}$ – check it's in scientific notation ✓

Recall that addition and subtraction of numbers with exponents (or indices) requires that the base and the exponent are the same. Since all numbers in scientific notation have the same base 10, for *addition* and *subtraction* calculations, we have to adjust the terms so the exponents are the same for both. This will ensure that the digits in the coefficients have the correct place value so they can be simply added or subtracted.

Here are the steps:

Addition	Subtraction
1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent	1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent
2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places	2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places
3. Add the new coefficients	3. Subtract the new coefficients
4. Convert the answer to scientific notation	4. Convert the answer to scientific notation
Example: $(3 \times 10^{2}) + (2 \times 10^{4})$ $4 - 2 = 2$ increase the small exponent by 2 to equal the larger exponent 4 0.03×10^{4} the coefficient of the first term is adjusted so its exponent matches that of the second term $= (0.03 \times 10^{4}) + (2 \times 10^{4})$ the two terms now have the same base and exponent and the coefficients can be added $= 2.03 \times 10^{4}$ check it's in scientific notation ✓	**Example:** $(5.3 \times 10^{12}) - (4.224 \times 10^{15})$ $15 - 12 = 3$ increase the small exponent by 3 to equal the larger exponent 15 0.0053×10^{15} the coefficient of the firs t term is adjusted so its exponent matches that of the second term $= (0.0053 \times 10^{15}) - (4.224 \times 10^{15})$ the two terms now have the same base and exponent and the coefficients can be subtracted. $= -4.2187 \times 10^{15}$ check it's in scientific notation ✓

Questions 3:

a) $(4.5 \times 10^{-3}) \div (3 \times 10^{2})$

b) $(2.25 \times 10^{6}) \times (1.5 \times 10^{3})$

c) $(6.078 \times 10^{11}) - (8.220 \times 10^{14})$ (give answer to 4 significant figures).

d) $(3.67 \times 10^{5}) \times (23.6 \times 10^{4})$

e) $(7.6 \times 10^{-3}) + (\sqrt{9.0 \times 10^{-2}})$

f) Two particles weigh 2.43 X 10^{-2} grams and 3.04 X 10^{-3} grams. What is the difference in their weight in scientific notation?

g) How long does it take light to travel to the Earth from the Sun in seconds, given that the Earth is 1.5 X 10^{8} km from the Sun and the speed of light is 3 X 10^{5} km/s?

4 Answers

Q1. Power Operations

a)

i. $5^2 \times 5^4 + 5^2 = 5^6 + 5^2$

ii. $x^2 \times x^5 = x^7$

iii. $4^2 \times t^3 \div 4^2 = t^3$

iv. $(5^4)^3 = 5^{12}$

v. $\frac{2^4 3^6}{3^4} = 2^4 3^2 = 16 \times 9 = 144$

vi. $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{27}$

vii. $\frac{9(x^2)^3}{3xy^2} = \frac{9x^6}{3xy^2} = \frac{3x^5}{y^2}$

viii. $a^{-1}\sqrt{a} = a^{-1} \times a^{\frac{1}{2}} = a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}\ or\ \frac{1}{a^{\frac{1}{2}}}$

b)

i. $x = 2$

ii. $x = -2$

iii. $x = 3$

iv. $x = 5$

v. Show that $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$ $\qquad \frac{\cancel{16}a^2b^3}{\cancel{3}a^3b} \times \frac{\cancel{9}a^3b^5}{\cancel{8}b^2a}$

$$= \frac{2a^5b^8 \times 3}{b^3a^4} = \frac{2a^1b^5 \times 3}{1} = 6ab^5$$

Q2. Scientific Notation

a) 4.5×10^2
b) 9.0×10^7
c) 3.5×10^0
d) 9.75×10^{-2}
e) 375
f) 39.7
g) 0.1875
h) 0.00875

Q3. Calculations with scientific notation

a) 1.5×10^{-5}
b) 3.375×10^9
c) -8.214×10^{14}
d) 8.8612×10^{10}
e) 3.076×10^{-2}
f) 2.126×10^{-2}
g) 500 s