Mathematics for Sports & Exercise Science

The module covers concepts such as:

- Maths refresher
- Fractions, Percentage and Ratios
- Unit conversions
- Calculating large and small numbers
- Logarithms
- Trigonometry
- Linear relationships

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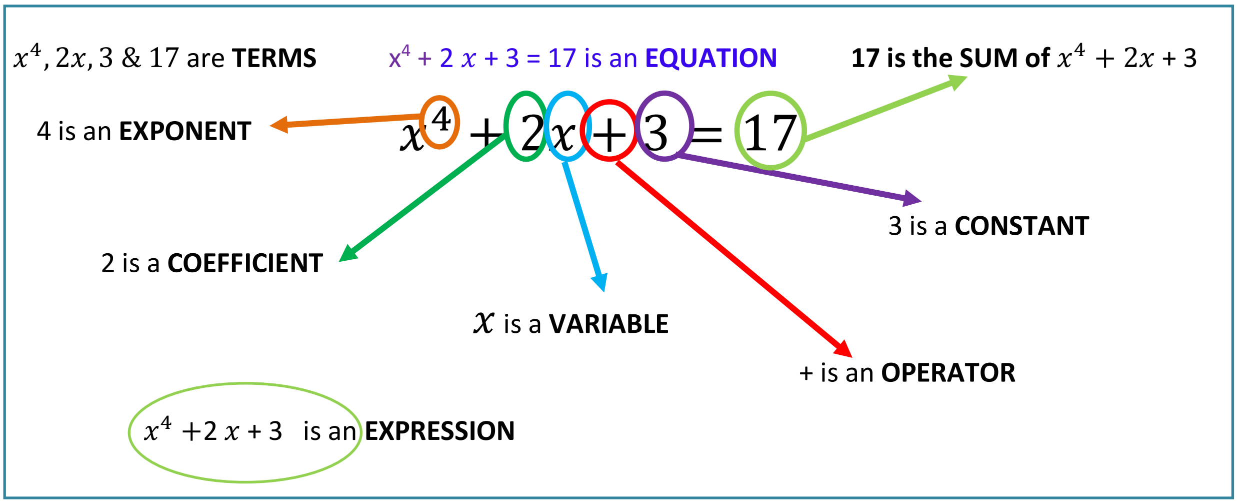
Mathematics for Sport and Exercise Science

Contents

1. Terms and Operations

Glossary



Equation: A mathematical sentence containing an equal sign. The equal sign demands that the expressions on either side are balanced and equal.

Expression: An algebraic expression involves numbers, operation signs, brackets/parenthesis and variables that substitute numbers but does not include an equal sign.

Operator: The operation $(+, -, \times, \div)$ which separates the terms.

Term: Parts of an expression separated by operators which could be a number, variable or product of numbers and variables. Eg. $2x$, 3 & 17

Variable: A letter which represents an unknown number. Most common is x, but can be any symbol.

Constant: Terms that contain only numbers that always have the same value.

Coefficient: A number that is partnered with a variable. The term $2x$ is a coefficient with variable. Between the coefficient and variable is a multiplication. Coefficients of 1 are not shown.

Exponent: A value or base that is multiplied by itself a certain number of times. Ie. x^4 represents $x \times x \times x \times x$ or the base value x multiplied by itself 4 times. Exponents are also known as Powers or Indices.

In summary:

Variable:	x	Operator:	$+$
Constant:	3	Terms:	3, $2x$ (a term with 2 factors) & 17
Equation:	$2x + 3 = 17$	Left hand expression:	$2x + 3$
Coefficient:	2	Right hand expression	17 (which is the sum of the LHE)

The symbols we use between the numbers to indicate a task or relationships are the **operators**, and the table below provides a list of common operators. You may recall the phrase, 'doing an operation.

Symbol	Meaning
$+$	Add, Plus, Addition, Sum
$-$	Minus, Take away, Subtract, Difference
$\times$	Times, Multiply, Product,
$\div$	Divide, Quotient
$\pm$	Plus and Minus
$\vert a\vert$	Absolute Value (ignore –ve sign)
$=$	Equal
$\neq$	Not Equal

Symbol	Meaning
$<$	Less than
$>$	Greater than
$\ll$	Much Less than
$\gg$	Much More than
$\approx$	Approximately equal to
$\leq$	Less than or equal
$\geq$	Greater than or equal
Δ	Delta
$\sum$	Sigma (Summation)

Order of Operations

The **Order of Operations** is remembered using the mnemonics BIDMAS or BOMDAS (Brackets, Indices or Other, Multiplication/Division, and Addition/Subtraction).

Brackets	{[( )]}
Indices or **O**ther	$x^2, \sin x, \ln x, \sqrt{}\ etc$
Multiplication or **D**ivision	$\times\ or\ \div$
Addition or **S**ubtraction	+ or -

The Rules:

1. Follow the order (BIMDAS, BOMDAS or BODMAS)
2. If two operations are of the same level, you work from left to right. E.g. $(\times\ or\ \div)\ or\ (+\ or\ -)$
3. If there are multiple brackets, work from the inside set of brackets outwards. {[( )]}

Example Problems:

1. Solve: $\frac{(3.4+4.6)^2}{\sqrt{(6+5\times2)}} - 8\times0.5$
2. Step 1: brackets first $= \frac{(8)^2}{\sqrt{16}} - 8\times0.5$
3. Step 2: then Indices $= \frac{64}{4} - 8\times0.5$
4. Step 3 then divide & multiply $= 16-4$
5. Step 4 then subtract $= 12$

Question 1:

a. $20-2^2\times5+1=$
b. $12\times-2+2\times7=$
c. $48\div6\times2-4=$
d. $\frac{0.3^2\times6}{0.2^2}+0.5=$
e. $\frac{(2+\sqrt{64})^3}{5^2\times4} - 10\times0.5=$

2. Percentage

The concept of percentage is an extension of fractions. To allow comparisons between fractions we need to use the same denominator. As such, all percentages use 100 as the denominator.

The word percent or "per cent" means **per 100**.

Therefore, 27% is $\frac{27}{100}$.

Items/measurements are rarely found in groups of exactly 100, so we can calculate the number of units that would be in the group if it did contain the exactly 100 units – the percentage.

Percent (%) = $\frac{number\ of\ units}{total\ number\ of\ units\ in\ the\ group} \times 100$

Example:

JCU has 2154 female and 1978 male students enrolled. What percentage of the students is female?

The total number of students is 4132, of which 2154 are female.

% female = $\frac{number\ of\ females}{total\ number\ of\ students} \times 100$

= $\frac{2154}{4132} \times 100$

= 52.13 %

Conversely, we might know a **percentage** change from a total, standard or starting value and can calculate what that change is in real units.

Example:

When John is exercising his heart rate rises to 180 bpm. His resting heart rate is 70 % of this. What is his resting heart rate?

70 % = $\frac{resting\ heart\ rate}{180} \times 100$

Rearrange:

Start with	70 % = $\frac{\boldsymbol{resting\ heart\ rate}}{\mathbf{180}} \times \mathbf{100}$
Divide both sides by 100	$\frac{\mathbf{70}}{\mathbf{100}} = \frac{\boldsymbol{resting\ heart\ rate}}{\mathbf{180}}$
Multiply both sides by 180	$\frac{\mathbf{70}}{\mathbf{100}} \times \mathbf{180}$ = resting heart rate

Resting heart rate = $\frac{70}{100} \times 180$

= 126 bpm

To use percentage in a calculation, the simple mathematical procedure is modelled below:

For example, 25% of 40 is $\frac{25}{100} \times 40 =$

Or $0.25 \times 40 = 10$

EXAMPLE:

For a person standing upright, the compressive force on both tibia is determined by the person's body mass, expressed as body weight in newtons. 88% of total body mass is above the knee, so how much compressive force acts on each tibia for a person with body weight 600 N?

Step 1: Since only 88% of body mass is exerting force, calculated 88% of the total 600 N

Step 2 $\frac{88}{100} \times 600\ N =$ or $0.88 \times 600\ N = 528\ N$

Step 3 This force is distributed to 2 tibia, so the force on one tibia is $\frac{528\ N}{2} = 264\ N$

Question 2:

a) Sally bought a television that was advertised for $467.80. She received a discount of $32.75. What percentage discount did she receive?

b) 50 kg of olives yields 23 kg of olive oil. What percentage of the olives' mass was lost during the extraction process?

c) The recommended daily energy intake for women is 8700 KJ. A woman is consuming only 85% of the recommended intake. What is her energy intake?

d) 56% of body weight is supported on the fifth lumbal vertebrae. For a body weight of 768 N, what is the compressional force on L5?

3. Algebra Refresh

Algebra provides a clear and descriptive way to explain the mathematical relationship between variables. It allows us to explore those relationships without the encumbrance of maniupulating unwieldy numbers, or if the actual values are unknown, to calculate unknown values from known relationships between variables.

When representing an unknown number using a variable, the choice of letter is not significant mathematically although a distinctive choice can aid memory. For example, "v" can be used to represent velocity.

Some basic algebra rules

Expressions with zeros and ones:
Zeros and ones can be eliminated. For example:

When zero is added the number isn't changed, $x + 0 = x$ or $x - 0 = x$

$$(6 + 0 = 6, \qquad 6 - 0 = 6)$$

If a number is multiplied by positive 1, the number stays the same, $x \times 1 = x$ or $\frac{x}{1} = x$

$$(6 \times 1 = 6, \ \ \frac{6}{1} = 6)$$

Note: Using indices (powers), any number raised to the power of zero is 1.

$$\frac{2^2}{2^2} = \frac{4}{4} = 1 \quad or \quad \frac{2^2}{2^2} = 2^{2-2} = 2^0 = 1$$

- Additive Inverse: $x + (-x) = 0$
- Any number multiplied by its reciprocal equals one. $x \times \frac{1}{x} = 1;\ 4 \times \frac{1}{4} = 1$
- Symmetric Property: $If\ x = y\ then\ y = x$
- Transitive Property: $If\ x = y\ and\ y = z, then\ x = z$
  For example, if apples cost $2 and oranges cost $2 then apples and oranges are the same price.

Remember the **Order of Operations -** BIDMAS or BOMDAS (Brackets, Order, Multiplication/Division, and Addition/Subtraction).

Also a **golden rule**:
"What we do to one side we do to the other"

Addition and Multiplication Properties

Maths Property	Rule	Example
Commutative **The number order for addition or multiplication doesn't affect the sum or product**	$a + b = b + a$ $ab = ba$	$1 + 3 = 3 + 1$ $2 \times 4 = 4 \times 2$
Associative **Since the Number order doesn't matter, it may be possible to regroup numbers to simplify the calculation**	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$	$1 + (2 + 3) = (1 + 2) + 3$ $2 \times (2 \times 3) = (2 \times 2) \times 3$
Distributive **A factor outside the bracket can be multiplied with individual terms within a bracket to give the same result**	$a(b + c) = ab + ac$	$2(3 + 1) = 2 \times 3 + 2 \times 1$
Zero Factor	$a \times 0 = 0$ If $ab = 0$, then either $a = 0$ or $b = 0$	$2 \times 0 = 0$
Rules for Negatives	$-(-a) = a$ $(-a)(-b) = ab$ $-ab = (-a)b = a(-b)$ $= -(ab)$ $(-1)a = -a$	$-(-3) = 3$ $(-2)(-3) = 2 \times 3$ $-2 \times 3 = (-2) \times 3$ $= 2 \times (-3)$ $= -(2 \times 3)$ $(-1) \times 2 = -2$
Rules for Division	$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ $\frac{-a}{-b} = \frac{a}{b}$ $If\ \frac{a}{b} = \frac{c}{d}\ then\ ad = bc$ $Proof:$ $\frac{a}{b} = \frac{c}{d}$ $\frac{\cancel{b} \times a}{\cancel{b}} = \frac{bc}{d}$ (multiply everything by b) $a = \frac{bc}{d}$ $a \times d = \frac{bc \times \cancel{d}}{\cancel{d}}$ (multiply by d) $ad = bc$	$-\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2}$ $\frac{-6}{-3} = \frac{6}{3}$ $If\ \frac{1}{2} = \frac{3}{4}\ then\ 1 \times 4$ $= 2 \times 3$

Additional help with algebra is available from the Algebra Basics module, downloadable from The Learning Centre.

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4. Collecting Like Terms

Algebraic thinking involves simplifying problems to make them easier to solve. The problem below consists of several related or "like" terms. Like terms can be grouped or collected, creating a smaller or simpler question.

$$7x + 2x + 3x - 6x + 2 = 14$$

A **like term** is a term which has the **same variable** (it may also have the same power/exponent/index), with only a **different coefficient**. In the equation above, there are four different coefficients $(7, 2, 3, \& 6)$ with the same variable, x, and no exponents to consider.

Like terms (multiplied by 'x') can be collected: $(\mathbf{7x} + \mathbf{2x} + \mathbf{3x} - \mathbf{6x}) + 2 = 14$
(+2 isn't a like term as it doesn't share the variable 'x')

The coefficients can be added and subtracted separate to the variables: $7 + 2 + 3 - 6 = 6$
Therefore: $7x + 2x + 3x - 6x = 6x$

The original equation simplifies to $6x + 2 = 14$
Now we solve the equation:

$$6x + 2(-2) = 14 - 2$$
$$6x = 12$$
$$6\,x(\div 6) = 12 \ \div 6$$
$$x = 2$$

Example Problem:

1. Collect the like terms and simplify:

$$5x + 3xy + 2y - 2yx + 3y^2$$

Step 1: Recognise the like terms (note: xy is the same as yx; commutative property)

$$5x + 3xy + 2y - 2yx + 3y^2$$

Step 2: Arrange the expression so that the like terms are together (remember to take the operator with the term).

$$5x + 2y + 3xy - 2yx + 3y^2$$

Step 3: Simplify the equation by collecting like terms: $5x + 2y + 1xy + 3y^2$
Note: a coefficient of 1 is not usually shown $\therefore$ $5x + 2y + xy + 3y^2$

Question 3

Simplify:

a. $3x + 2y - x$

b. $3m + 2n + 3n - m - 7$

c. $2x^2 - 3x^3 - x^2 + 2x$

Expand the brackets then collect like terms

d. $3(m + 2n) + 4(2m + n)$

e. $4(x + 7) + 3(2x - 2)$

5. Solving Equations

The equal sign of an **equation** indicates that both sides of the equation are equal. The equation may contain an **unknown quantity** (or variable) whose value can be calculated. In the equation, $5x + 10 = 20$, the unknown quantity is x. This means that 5 multiplied by something (x) and added to 10, will equal 20.

- To solve an equation means to find all values of the unknown quantity so that they can be substituted to make the left side and right sides **equal**
- Each such value is called a **solution** (e.g. $x = 2$ in the equation above)
- The equation is rearranged and solved in reverse order of operation (SADMOB – see page 4) to find a value for the unknown

  Eg. $5x + 10 = 20$ **First rearrange by subtracting 10 from the LHS and the RHS**

  $5x + 10\,(-10) = 20(-10)$

  $5x = 10$

  $\frac{5x}{5} = \frac{10}{5}$ **Divide both LHS and RHS by 5**

  $x = 2$

  To check the solution, substitute $x\ for\ 2$; $(5 \times 2) + 10 = 20 \therefore 10 + 10 = 20$ ✓

Four principles to apply when solving an equation:

1. **Work towards solving the variable**: ($x =$)
2. **Use the opposite mathematical operation**: Remove a constant or coefficient by doing the opposite operation on both sides:

   Opposite of $\times\ is\ \div$

   Opposite of $+\ is\ -$

   Opposite of $x^2\ is\ \sqrt{x}$

   Opposite of $\sqrt{x}\ is\ \pm x^2$
3. **Maintain balance**: "What we do to one side, we must do to the other side of the equation."
4. **Check:** Substitute the value back into the equation to see if the solution is correct.

ONE-STEP EQUATIONS

Addition	Subtraction
$x + (-5) = 8$ $x + (-5) - (-5) = 8 - (-5)$ $x = 8 + 5$ $\therefore x = 13$	$x - 6 + 6 = (-4) + 6$ So $x = (-4) + 6$ $\therefore x = 2$
Check by substituting 13 for x $13 + (-5) = 8$ $13 - 5 = 8$ $8 = 8$ ✓	Check by substituting 2 for x $2 - 6 = (-4)$ ✓ $-4 = -4$

Question 4

Solve for x:

$a)\ x + 6 - 3 = 18$

$b)\ 7 = x + (-9)$

$c)\ x - 12 = (-3)$

d) $18 - x = 10 + (-6)$

Two-step Equations

The following equations require two steps to single out the variable (i.e. To solve for x).

Example: $2x + 6 = 14$

Step 1: The constant 6 is subtracted from both sides, creating the following equation:

$$2x + 6 - 6 = 14 - 6 \quad \textbf{(The opposite of +6 is −6)}$$
$$2x = 8$$

Step 2: Next both sides are divided by two, creating the following equation:

$$\frac{2x}{2} = \frac{8}{2} \quad \textbf{(The opposite of } 2x \textbf{ is } \div 2\textbf{)}$$
$$\therefore x = 4$$

Check. Substitute x = 4 into the equation: $2 \times 4 + 6 = 14$

$$8 + 6 = 14$$
$$14 = 14 \quad ✓ \quad \textbf{(The answer must be correct)}$$

Multi-step example:

Solve for *T*: $\frac{3T}{12} - 7 = 6$

Set your work out in logical clear to follow steps

$$\frac{3T}{12} - 7 = 6$$
$$\frac{3T}{12} - 7 + 7 = 6 + 7$$
$$\frac{3T}{12} = 13$$
$$\frac{3T}{12} \times 12 = 13 \times 12$$
$$3T = 156$$
$$\frac{3T}{3} = \frac{156}{3}$$
$$\therefore T = 52$$

Check: $(3 \times 52) \div 12 - 7 = 6$ ✓

Question 5:

Solve the following to calculate the unknown variable:

Solve for x:

a. $5x + 9 = 44$

b. $\frac{x}{9} + 12 = 30$

c. $8 - \frac{x}{11} = 30$

Solve for $\overrightarrow{v_1}$:

d) $\overrightarrow{v_2} = \overrightarrow{v_1} + \vec{a}t$

Solve for $\vec{x}$

e) $\overrightarrow{v_2}^2 = \overrightarrow{v_1}^2 + 2\vec{a}\vec{x}$

8. Rearranging Formulas

A formula uses symbols and rules to describe a relationship between quantities. In mathematics, formulas follow the standard rules for algebra and can be rearranged as such. If values are given for all other variables described in the formula, rearranging allows for the calculation of the unknown variable. This is a mathematical application of working with variables and unknowns in physics and kinematic problems. In this section, rearranging equations and substituting values is practiced with common physics formulas to determine unknowns. The next section applies these skills to problem solving including application of appropriate units.

Basic Rearranging Example:

Calculate the density $(\boldsymbol{\rho})$ of apatite

Given: Mass (m) = 1.595 Volume (v) = 0.5

$\boldsymbol{\rho} = \frac{m}{V}$ [density is defined by this relationship]

$$\boldsymbol{\rho} = \frac{1.595}{0.5}$$

$$\boldsymbol{\rho} = 3.19$$

Alternatively, calculate the Mass (m) of Lithium

Given: Density (ρ) = 3.19 and Volume (V) = 0.5

$$\rho = \frac{\boldsymbol{m}}{V}$$

$V \times \rho = \frac{\boldsymbol{m} \times \cancel{V}}{\cancel{V}}$ (rearrange to calculate m)

$$V\rho = \boldsymbol{m}$$

$\boldsymbol{m} = 0.5 \times 3.19$ (substitute values and calculate)

$$\boldsymbol{m} = 1.595$$

Finally, calculate the Volume (V) of Lithium

Given: Density (ρ) = 3.19 and Mass (m) = 1.595

$$\rho = \frac{m}{\boldsymbol{V}}$$

$\boldsymbol{V}\rho = m$ (as above)

$$\frac{\boldsymbol{V}\cancel{\rho}}{\cancel{\rho}} = \frac{m}{\rho}$$

$$\boldsymbol{V} = \frac{m}{\rho}$$

$\boldsymbol{V} = \frac{1.595}{3.19}$ = 0.5

Many other equations use exactly the same process of rearranging

For example: $\vec{a} = \frac{\Delta \vec{v}}{t}$ $\vec{F} = m\vec{a}$ $P = \frac{F}{A}$ $v = \frac{d}{t}$ $M = mv$

And $J = Ft$

Note that each formula describes a relationship between values:

$a = acceleration\ vector$
$v = acceleration\ vector$
$\vec{a} = acceleration\ vector$
$\vec{v} = velocity\ vector$
$t = time$
$\vec{F} = force$
$m = mass$
$P = pressure$
$A = area$
$d = displacement$
$T = torque$
$t = time$
$M = momentum$
$J = joules$

Question 6:

a. Using $v = \frac{d}{t}$

i. Given d = 50 and t = 2, calculate v

ii. Given v = 60 and d = 180, rearrange the equation to calculate t

iii. Given v = 100 and t = 4, rearrange the equation to calculate d

b. Using $\vec{F} = m\vec{a}$

i. Given m = 12 and $\vec{a}$ = 4, calculate $\vec{F}$

ii. Given $\vec{F}$= 81 and $\vec{a}$ =3, calculate m

iii. Given $\vec{F}$= 48 and calculate m=12, calculate $\vec{a}$

c. Using $\vec{F}\Delta t = \Delta\vec{p}$

i. Given Δt = 5 and $\Delta\vec{p}$ = 50, rearrange the equation to calculate $\vec{F}$

ii. Given $\vec{F}$ = 5 and $\Delta\vec{p}$ = 30, rearrange the equation to calculate Δt

d. Using $\vec{a} = \frac{\Delta\vec{v}}{t}$

i. Given $\Delta\vec{v}$ = 60 and t = 12, calculate $\vec{a}$

ii. Given $\Delta\vec{v}$ = 124 and $\vec{a}$ = 4 , calculate t

e. Using $\vec{x} = \frac{\vec{v_1}+\vec{v_2}}{2} \times t$

i. Given $\vec{v_1}$ = 15 , $\vec{v_2}$ = 25 and t = 10, calculate $\vec{x}$

ii. Given $\vec{v_1}$ = 60 , $\vec{v_2}$ = 90 and $\vec{x}$ = 225, calculate t

f. Using ${v_2}^2 = {v_1}^2 + 2ax$

i. Given v_1 = 9 , a = 6 and x = 12, calculate v_2

ii. Given v_2 = 10 , a = 8, x = 4, calculate v_1

iii. Given v_2 = 12 , a = 8, v_1 = 8, calculate x

6. Power Operations

Powers are also called **exponents** or **indices**; we can work with **indices** to simplify expressions and to solve problems.

Some key ideas:

a) Any base number raised to the power of 1 is the base itself: for example, $5^1 = 5$
b) Any base number raised to the power of 0 equals 1, so: $4^0 = 1$
c) Powers can be simplified if they are **multiplied** or **divided** and have the **same** base.
d) Powers of powers are multiplied. Hence, $(2^3)^2 = 2^3 \times 2^3 = 2^6$
e) A negative power indicates a reciprocal: $3^{-2} = \frac{1}{3^2}$

Certain rules apply and are often referred to as: **Index Laws.**

Below is a summary of the index rules:

Index Law	Substitute variables for values
$\boldsymbol{a^m \times a^n = a^{m+n}}$	$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
$\boldsymbol{a^m \div a^n = a^{m-n}}$	$3^6 \div 3^3 = 3^{6-3} = 3^3 = 27$
$\boldsymbol{(a^m)^n = a^{mn}}$	$(4^2)^5 = 4^{2\times 5} = 4^{10} = 1048\,576$
$\boldsymbol{(ab)^m = a^m b^m}$	$(2 \times 5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100$
$\boldsymbol{({}^{a}/_{b})^m = a^m \div b^m}$	$(10 \div 5)^3 = 2^3 = 8;\ (10^3 \div 5^3) = 1000 \div 125 = 8$
$\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$\boldsymbol{a^{\frac{1}{m}} = \sqrt[m]{a}}$	$8^{1/3} = \sqrt[3]{8} = 2$
$\boldsymbol{a^0 = 1}$	$6^3 \div 6^3 = 6^{3-3} = 6^0 = 1; (6 \div 6 = 1)$

EXAMPLE PROBLEMS:

a) Simplify $6^5 \times 6^3 \div 6^2 \times 7^2 + 6^4 =$

$$= 6^{5+3-2} \times 7^2 + 6^4$$

$$= 6^6 \times 7^2 + 6^4$$

b) Simplify $g^5 \times h^4 \times g^{-1} =$

$$= g^5 \times g^{-1} \times h^4$$

$$= g^4 \times h^4$$

KHANACADEMY

Watch this short Khan Academy video for further explanation:
"Simplifying expressions with exponents"
https://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/simplifying-expressions-with-exponents

Question 7:

a. *Apply the index laws/rules:*

i. Simplify $5^2 \times 5^4 + 5^2 =$

ii. Simplify $x^2 \times x^5 =$

iii. Simplify $4^2 \times \mathrm{t}^3 \div 4^2 =$

iv. Simplify $(5^4)^3 =$

v. Simplify $\frac{2^4 3^6}{3^4} =$

vi. Simplify $3^2 \times 3^{-5} =$

vii. Simplify $\frac{9(x^2)^3}{3xy^2} =$

viii. Simplify $a^{-1}\sqrt{a} =$

b. What is the value of x for the following?

i. $49 = 7^x$

ii. $\frac{1}{4} = 2^x$

iii. $88 = 11^1 \times 2^x$

iv. $480 = 2^x \times 3^1 \times 5^1$

v. Show that $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$

7. Working with Units

In real world mathematical applications, physical quantities include units. Units of measure (eg. metres, Litres or kilograms) describe quantities based on observations or measurement. There are seven base units in the International Systme of Units (SI).

Other units are derived from these base units. Derived units often describe mathematical relationships between measurements of different properties. For example, force descrit

Base SI Units and some derived units

Base Units		
Base Unit Name	**Unit Symbol**	**Quantity**
Metre	m	Length
Kilogram	kg	Mass
Second	s	Time
Ampere	A	Electric Current
Kelvin	K	Temperature
Mole	mol	Amount of a Substance
Candela	cd	Luminous Intensity

Some Derived Units		
Derived Unit Name		**Quantity**
newton	$kg.m.s^{-2}$	Force, weight
pascal	$kg\cdot m^{-1}\cdot s^{-2}$	pressure
metre per second	$m\cdot s^{-1}$	velocity
metre per second squared	$m\cdot s^{-2}$	acceleration
newton metre	$m^2\cdot kg\cdot s^{-2}$	torque
newton second	$m\cdot kg\cdot s^{-1}$	momentum, impulse
newton metre second	$m^2\cdot kg\cdot s^{-1}$	angular momentum

Converting units of measure may mean converting between different measuring systems, such as from imperial to metric (e.g. mile to kilometres), or converting units of different scale (e.g. millimetres to metres).

Unit Conversions - Two Methods:

Method 1: Base Unit Method

Most formulas are written using base units (or derived units), meaning any values in the correct base unit form can be used. However, measured values may not be in the same unit as the base units of the formula. In these cases, the measured value can be converted to the base unit using a conversion factor.

Example:

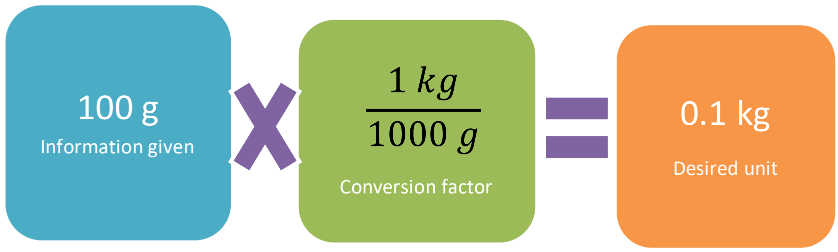
The specific heat of gold is 129 $\frac{J}{kg \cdot K}$. *What is the quantity of heat energy required to raise the temperature of 100 g of gold by 50 K?*

The solution is derived from the specific heat formula: $\mathrm{Q = mc\Delta T}$

Q=heat energy (base unit Joules, J)
m = mass **(base unit kilograms, kg)**
c = specific heat (derived unit $\frac{J}{kg \cdot K}$)
ΔT = change in temperature (T) (base unit Kelvin, K)

All the given information is in the correct base unit *except* mass (g) which **must be converted** to the SI base unit kg. We know that 1kg = 1000 g so this is the conversion factor:

m=



$$m = 100g \times \frac{1kg}{1000g} = 0.1kg$$

The unit for specific heat & temperature are already in their base SI unit and do not require conversion.

Applying the values to the variables in the equation gives:

$Q = 0.1 \times 129 \times 50$

Solution:

$Q = 645J$ (Joules is the base SI unit for heat energy)

∴ The quantity of heat required to raise the temperature of 100g of gold to 50K is 645J

Question 8

Convert the following:

a) Convert 120 g to kilograms.

b) Convert 4.264 L to kilolitres and millilitres

c) Convert 670 micrograms to grams.

d) How many grams (water) are in a cubic metre? (1 cm^3 = 1 g). How many kg?

e) How many inches in 38.10cm (2.54cm = 1 inch)

f) How many centimetres in 1.14 kilometres?

g) How many seconds are in 0.2 hours?

Method 2: Factor Label Method

Expressing a formula with units and working with units algebraically (or "dimensional analysis") has several advantages, especially when working with non-standard units. Units can be converted, the change from initial units to final unit is immediately clear and the formula may even be derived directly from the final unit. Algebraic methods are applied to both the variables and their units, allowing for the substitution and cancellation of units.

Unit Conversion Example:

Convert 10 km per hour into metres per second.

This question can be split into 3 parts: find a conversion factor for kilometres to metres, find a conversion factor for hours to seconds, and multiply the initial value by the conversion factors to find a final value in correct units.

$$\boldsymbol{Initial\ Value\ \times Conversion\ Factors = Final\ Value}$$

1. The relationship between kilometres and metres is 1km = 1000m. This is used to create a conversion factor.
    a. 10 km/hr is written as a unit fraction (10 km is travelled per 1 hr)
    $\frac{\mathbf{10}\boldsymbol{km}}{\mathbf{1}\boldsymbol{hr}}$
    b. determine the relationship of the units in the conversion factor:
    $\frac{\mathbf{10}\ \boldsymbol{km}}{\mathbf{1}\ \boldsymbol{hr}} \times \frac{}{\boldsymbol{km}}$ **Converting from km, so written on bottom of the conversion factor they will cancel out**
    c. The desired final unit is then written to complete the fraction
    $\frac{\mathbf{10}\ \boldsymbol{km}}{\mathbf{1}\ \boldsymbol{hr}} \times \frac{\boldsymbol{m}}{\boldsymbol{km}}$ **Converting to m, so write as the numerator**
    d. The conversion factor is made to equal 1 using the relationship between units.
    $\frac{\mathbf{10}\ \boldsymbol{km}}{\mathbf{1}\ \boldsymbol{hr}} \times \frac{\mathbf{1000}\ \boldsymbol{m}}{\mathbf{1}\ \boldsymbol{km}}$ **Remember:** $\frac{1km}{1000m}\ or\ \frac{1000m}{1km} = 1$

2. The relationship between hours and minutes is 1hour = 60 min, and the relationship between minutes and seconds is 1min = 60 sec. This conversion will take 2 steps and create 2 conversion factors.
   a. 10km/hr is written as a unit fraction
   $\frac{10\ km}{1\ hr}$
   b. The unit being converted is multiplied on the opposite side of the conversion factor fraction
   $\frac{10\ km}{1\ hr} \times \frac{hr}{}$
   c. The desired final unit is written to complete the conversion factor fraction
   $\frac{10\ km}{1\ hr} \times \frac{hr}{min}$
   d. The conversion factor is made to equal 1 using the relationship between units..
   $\frac{10\ km}{1\ hr} \times \frac{1\ hr}{60\ min}$
   e. The process is repeated to convert minutes to seconds
   $\frac{10\ km}{1hr} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ sec}$

3. The initial value of 10km/hr is then multiplied by the conversion factors:
$\frac{10\ km}{1\ hr} \times \frac{1000\ m}{1\ km} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ sec}$

4. Values and units appearing on both the top and bottom of the division cancel out.

$\frac{10\ \cancel{km}}{\cancel{1\ hr}} \times \frac{1000\ m}{\cancel{1\ km}} \times \frac{\cancel{1\ hr}}{60\ \cancel{min}} \times \frac{\cancel{1\ min}}{60\ sec}$

5. The remaining values and units are multiplied and divided following normal conventions.
$\frac{10 \times 1000\ m}{60 \times 60\ sec} = \frac{10000\ m}{3600\ sec} = \frac{100\ m}{36\ sec} = 2.78\ m/sec$ = 2.78 m/s

Solving the Equation Example:

A baby elephant walks at a constant velocity of 3.6 km/hr having a kinetic energy of 14.125 J. What is the mass of the baby elephant?

Approaching the problem using **GRASP**:

<u>**G**iven</u> (and Gather):

Variables -
E_k = Kinetic Energy = 14.125 J
v = velocity = 2 km/hr

The formula for kinetic energy is: $E_k = \frac{mv^2}{2}$

This formula is defined by the following units for each variables:

E_k = kinetic energy in Joules = $\boldsymbol{kg\ m^2\ s^{-2}}$ or $\frac{\boldsymbol{kg \times m^2}}{\boldsymbol{s^2}}$

v = velocity in units m s^{-1} or $\frac{m}{s}$

m = mass in units kg

R<u>equired</u>:

The required answer is the mass of the baby elephant. The formula can be rearranged algebraically to give a solution for mass using the variables given:

$$E_k = \frac{mv^2}{2}$$

$$E_k 2 = mv^2$$

$$\frac{E_k 2}{v^2} = m$$

A<u>nalysis</u>

Known values and their units can be substituted into the formula:

$$m = \frac{14.125\ \boldsymbol{kg\ m^2\ s^{-2}} \ \times 2}{(2\ km\ hr^{-1})^{\ 2}}$$

$$\frac{2\ \cancel{km}}{\cancel{hr}} \times \frac{1000\ m}{1\ \cancel{km}} \times \frac{1\ \cancel{hr}}{3600\ s}$$

Units m and km are both units of distance but incompatible with each other. Km must be converted to m; hr must be converted to s

Use the two conversion factors (1km = 1000m; 1 hr = 3600 s) and multiply out

$$m = \frac{14.125\ \boldsymbol{kg\ m^2\ s^{-2}} \ \times 2}{(\frac{2000\ m}{3600\ s})^{\ 2}}$$

Show the full equation with the converted units

$$m = \frac{14.125\ \boldsymbol{kg\ m^2\ s^{-2}} \ \times 2}{(0.556\ m\ s^{-1})^{\ 2}}$$

Can also be written as this

$$m = \frac{28.250\ \boldsymbol{kg}\ \cancel{\boldsymbol{m^2}}\ \cancel{\boldsymbol{s^{-2}}}}{0.309\ \ \cancel{m^2}\cancel{s^{-2}}}$$

Expand the denominator and cancel units; multiply out the values

$$m = 91.4\ kg$$

Calculate the final value with the remaining unit of kg.

S<u>olution</u>

Check that the units of the calculated value are appropriate for the required solution.

We were asked to find mass, and the unit for mass is Kg, so our answer is correct.

P<u>araphrase</u>

The mass of the baby elephant is 91.4 kg.

Question 9:

a. Convert 250 g/cm^3 to kg/m^3

b. What is 7.2 km/hr in metres per second?

c. When a 500 g ball is kicked, it accelerates at 432 km/hr. What was the force of the kick?

KHANACADEMY

Watch this short Khan Academy video for further explanation on converting units:

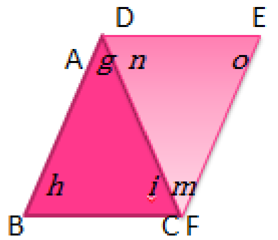
"Introduction to dimensional analysis"

https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:working-units/x2f8bb11595b61c86:rate-conversion/v/dimensional-analysis-units-algebraically

8. Trigonometry

Trigonometry is the study of the properties of triangles, as the word suggests. Considering that all polygons can be divided into triangles, understanding properties of triangles is important. Trigonometry has applications for a range of science and engineering subjects.

Congruence



If two plane shapes can be placed on top of each other exactly, then they are **congruent.** The corresponding **angles**, for example, $\angle BAC \text{ and } \angle DFE$ $(g \text{ and } m)$ are the same, and the **intervals,** for instance, $AB \text{ and } FE$ of each shape match perfectly. In addition, the perimetre and the area will be identical, thus the perimetre and area of ΔABC is identical to the perimetre and area of ∇DEF. Also, the vertices and sides will match exactly for ΔABC & ∇DEF.

In summary: $g = m; \quad h = o; \quad i = n$

$AB = EF; \quad BC = ED; \quad AC = FD$

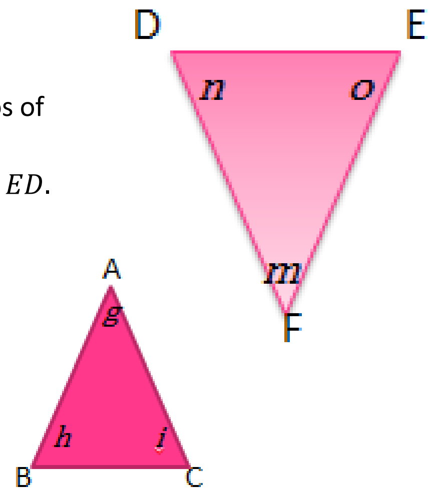
The mathematical symbol for congruence is " $\cong$ ". Therefore, $\Delta ABC \cong \nabla DEF$

Many geometric principles involve ideas about congruent triangles.

Similar

If a shape looks that same but is a different size it is said to be **similar**. The corresponding angles in shapes that are similar will be **congruent**. The ratios of adjacent sides in the corresponding angle will be the same for the ratio of adjacent sides in a similar triangle. For example: the ratio $AB{:}\,BC \text{ is as } EF{:}\,ED$.

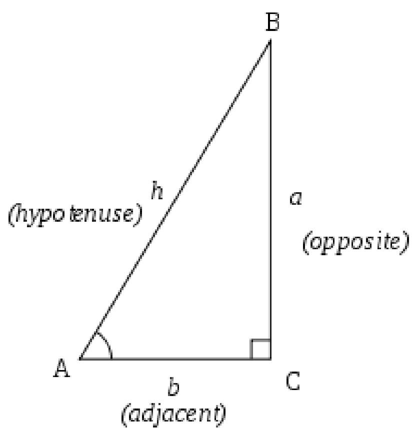
The angles are the same: $g = m; \quad h = o; \quad i = n$



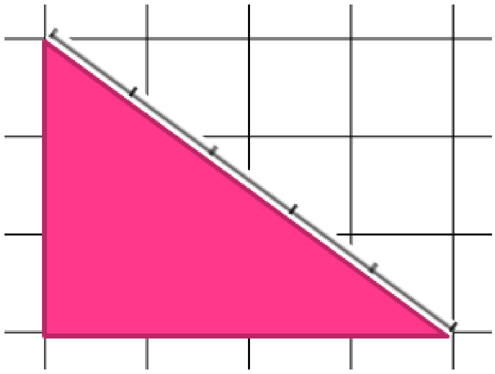
Right Angle Triangles and the Pythagorean Theorem

To begin, we can identify the parts of a right angled triangle:

- The right angle is symbolised by a **square**.
- The side directly opposite the right angle is called the **hypotenuse**. The hypotenuse **only** exists for right angle triangles.
- The hypotenuse is always the longest side.
- If we focus on the angle ∠ BAC, the side opposite is called the **opposite** side.
- The side touching angle ∠ BAC is called the **adjacent** side.



Investigating right angle triangles



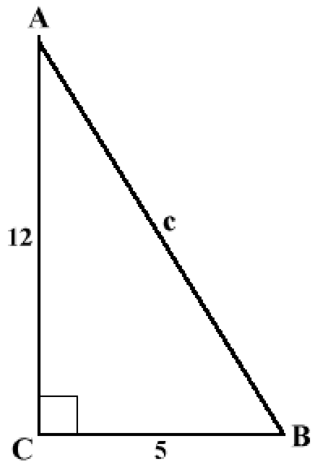
The **hypotenuse** is related to a Greek word that means to stretch. What do you do if you were to make a right angle and you did not have many measuring instruments? One way is to take a rope, mark off three units, turn; then four units, turn; and then five units. Join at the beginning and when stretched out, you should have a triangle. The triangle created will form a right angle, as right.

The relationship between these numbers, 3, 4 and 5 were investigated further by Greek mathematicians and are now commonly known now as a Pythagorean triple. Another triple is 5, 12 and 13. You might recall the mathematical formula: $\boldsymbol{a^2 + b^2 = c^2}$. This is one of the rules of trigonometry; the Pythagoras theorem which states:

"The square of the hypotenuse is equal to the sum of the square of the other two sides"

Example Problems:

1. Calculate the length of the unknown side.



Step 1: Recognise the hypotenuse is the unknown c.

Step 2: Write the formula: $c^2 = a^2 + b^2$

Step 3: Sub in the numbers $c^2 = 5^2 + 12^2$

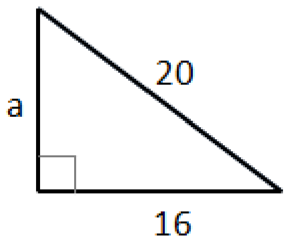
$c^2 = 25 + 144$

$c^2 = 169$

$c = \sqrt{169}$ (The opposite of square is root)

$c = 13$

2. Calculate the length of the unknown side.



Step1: Recognise the adjacent side is unknown a.

Step 2: Write the formula: $c^2 = a^2 + b^2$

Step 3: Sub in the numbers $20^2 = a^2 + 16^2$

$400 = a^2 + 256$

$400 - 256 = a^2$ (Rearrange)

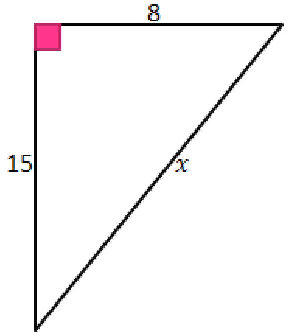
$144 = a^2$

$\sqrt{144} = a$

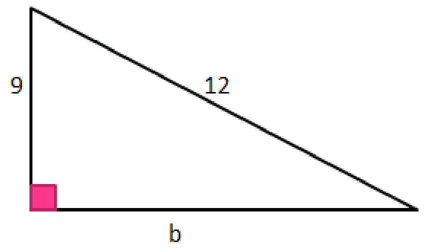
$12 = a$

Question 10:

a. Calculate the unknown side x :



b. Calculate the unknown side b correct to two decimal places:

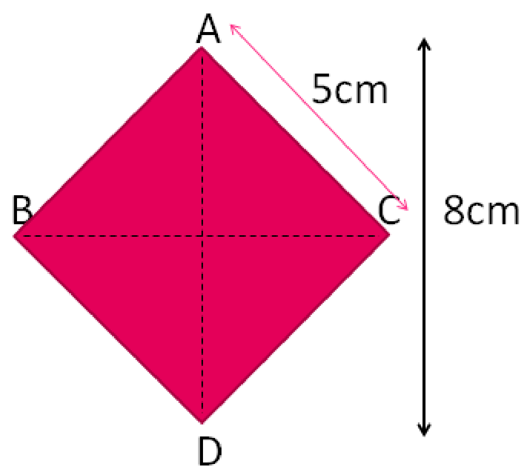


c. A support wire is required to strengthen the sturdiness of a pole. The pole stands 20m and the wire will be attached to a point 15m away. How long will the wire be?

d. Here we have a rhombus $ABCD$. It has a diagonal of 8cm. It has one side of 5cm.

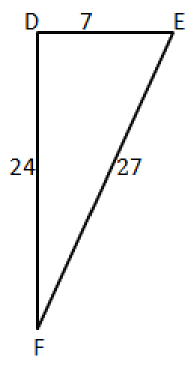
i) Find the length of the other diagonal (use Pythagoras Theorem)

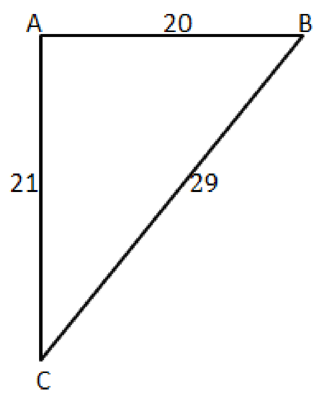
ii) Find the area of the rhombus.



e. Which of the following triangles is a right angle triangle? Explain why.

f. Name the right angle.

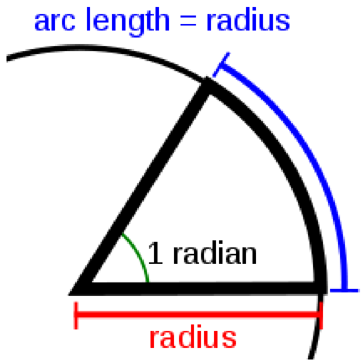




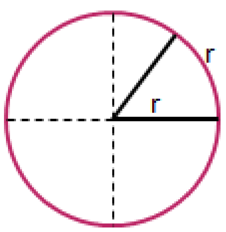
Measuring Angles: Radians and Degrees

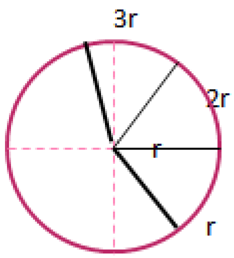
Angles are often measured in two ways, degrees and radians. An angle is measured by the amount of turn of a line. The amount of turn relates to the circle, where a full revolution is 360°. Hence, 1 **degree** $(1°)$ is $\frac{1}{360}th$ of a full revolution. If we take a line section AB, and rotate it half a revolution $(\frac{180°}{360°})$ to the position of AC, then we get a straight angle as shown.

A **radian,** which is short for **radius angle**, is also based on the concept of a circle. If the arc length of a sector is equal to the radius, then we can say that the angle is 1 radian. If the angle is in degrees, we must use the correct symbol ′ ° ′ to show that the angle has been measured in degrees. Otherwise it is assumed that the angle is measured in radians. Often radian is abbreviated, so 1 radian will be abbreviated to 1.



The images below show that an arc length of $1r$ is opposite to (subtends) an angle of 1 radian. Then in the next image we can see that an arc length of $3r$ subtends an angle of 3radians.





Thus if we think about the circumference of a circle as $C = 2\pi r$, we could say that an arc of $2\pi r$ subtends the radius angle of 2π. In other words, the circumference of a circle subtends a full revolution. This then implies that radius angle of a circle is $2\pi = 360° \therefore \pi = 180°$.

EXAMPLE PROBLEMS:

Convert to radians:

1. $Convert\ 30°\ into\ radians$

$180° = \pi$ and thus $1° = \frac{\pi}{180}$

$30° = 30^0 \times \frac{\pi}{180^0}$

$\therefore 30° = \frac{\pi}{6} \approx (0.52)$

2. $Convert\ 90°\ into\ radians$

$90° = 90 \times \frac{\pi}{180}$

$\therefore 90° = \frac{\pi}{2} \approx (1.57)$

Convert to degrees:

1. $Convert\ 1\ radian\ to\ degrees$

So $\pi = 180°$

$1 = \frac{180^0}{\pi} \approx 57°$

$\therefore 1\ radian \approx 57°$

2. $Convert\ 3\ radian\ to\ degrees$

$3 = 3 \times \frac{180}{\pi} \approx 172°$

$\therefore 3\ radian \approx 172°$

Note: to convert radian to degrees we can apply the formula: $1\ Radian = \frac{180°}{\pi} \approx 57°$

Questions 11:

a. Convert to 72° into radians

b. Convert 0.7π in degrees

Trigonometric Functions: Sine, Cosine and Tangent

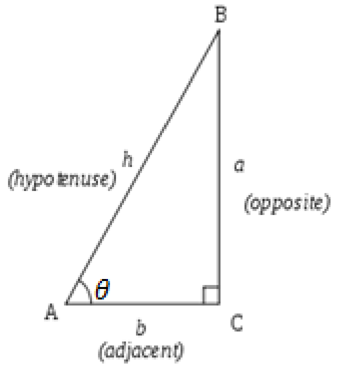
In this section we extend on the Pythagoras Theorem, which relates to the three sides of a right angled triangle, to trigonometrical ratios, which help to calculate an angle of a triangle involving lengths and angles of right angle triangles. This is a basic introduction to trigonometry that will help you to explore the concept further in your studies.
Often angles are marked with $'\ \theta\ '$ which is the Greek letter 'theta'. This symbol helps to identify which angle we are dealing with.

For instance, to describe the right angle triangle (right)

The side BC is opposite θ

The side AC is adjacent θ

The opposite side of the right angle is the hypotenuse AB



The trigonometrical ratios do not have units of measure themselves – they are ratios. The three ratios are cosine of θ, the sine of θ and the tangent of θ. These are most often abbreviated to sin, cos and tan.

We can calculate an angle when given one of its trigonometrical ratios.
The ratios depend on which angles and sides are utilised
The three ratios are:

- $\sin A = \dfrac{opposite}{hypotenuse}$
- $\cos A = \dfrac{adjacent}{hypotenuse}$
- $\tan A = \dfrac{opposite}{adjacent}$

If you are required to work with these ratios, you might like to memorise the ratios as an acronym SOHCAHTOA pronounced "sock – a- toe – a" or the mnemonic: "**S**ome **O**ld **H**umans **C**an **A**lways **H**ide **T**heir **O**ld **A**ge."
The formulas can be used to either find an unknown side or an unknown angle.

Example Problems:

1. Find x

Step 1: Label the triangle: $19m$ is the hypotenuse and x is adjacent to the 60°

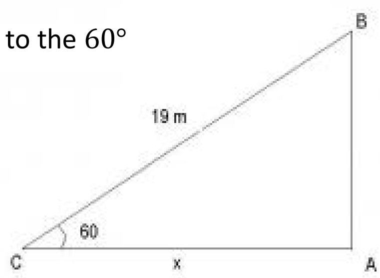
Step 2: Write the correct formula: $\cos A = \dfrac{adjacent}{hypotenuse}$

Step 3: Sub in the numbers: $\cos 60 = \dfrac{x}{19}$

Step 4: Rearrange the formula: 19 cos 60 = x

Step 5: Enter into calculator: 9.5 = x

Therefore, Side x = 9.5 m



2. Find a

Step 1: Label the triangle: 6.37in is opposite and 15.3in is adjacent to the angle

Step 2: Write the correct formula: $\tan A = \frac{opposite}{adjacent}$

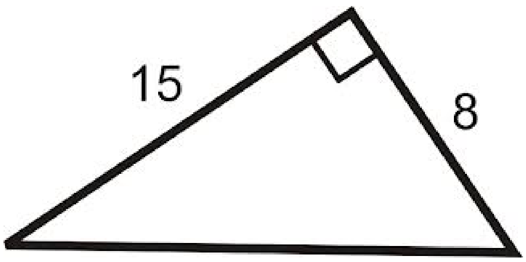
Step 3: Sub in the numbers: $\tan a = \frac{6.37}{15.3}$

Step 4: Rearrange the formula: $a = \tan^{-1}\left(\frac{6.37}{15.3}\right)$

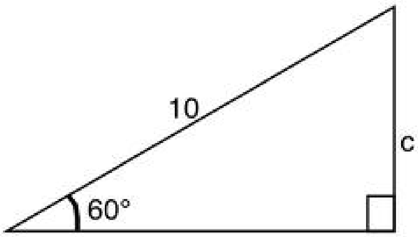
Step 5: Enter into calculator: $a = 22.6^{o}$

Question 12:

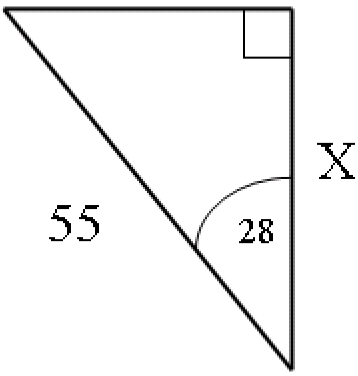
a) Find both angles in the triangle below.



b) Find C in the triangle below.



c) Find X in the triangle below.



Applying Trigonometric Functions

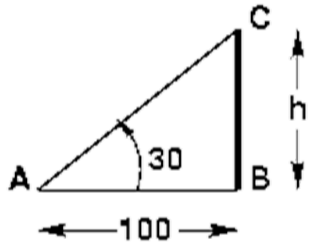
Trigonometric Functions are used in a wide range of professions to solve measurement problems, e.g. architecture, cartography, navigation, land-surveying and engineering. Less obvious uses include the study of distances between stars in astronomy and more abstract applications in geophysics, medical imaging, seismology and optics. The sine and cosine functions are particularly important to the theory of periodic functions such as those that describe sound and light waves.

To simulate the real world application of trigonometric functions, you may be asked to solve word problems like the ones below in your exams.

EXAMPLE PROBLEMS:

1) If the distance of a person from a tower is 100 m and the angle subtended by the top of the tower with the ground is 30°, what is the height of the tower in metres?

Step 1:
Draw a simple **diagram** to represent the problem. Label it carefully and clearly mark out the quantities that are given and those which have to be calculated. Denote the unknown dimension by say h if you are calculating height or by x if you are calculating distance.



Step 2:
Identify which trigonometric function represents a ratio of the side about which information is given and the side whose dimensions we have to find out. Set up a **trigonometric equation**.

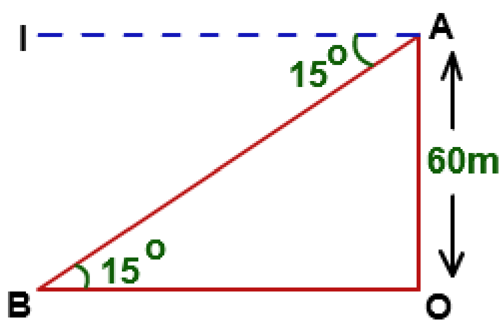
AB = distance of the man from the tower = 100 m
BC = height of the tower = h (to be calculated)
The trigonometric function that uses AB and BC is tan a , where a = 30^0

Step 3:
Substitute the value of the trigonometric function and **solve the equation** for the unknown variable.

$$\tan 30^0 = \frac{BC}{AB} = \frac{h}{100m}$$
$$h = 100m \, x \tan 30^0 = 57.74m$$

2) From the top of a light house 60 metres high with its base at the sea level, the angle of depression of a boat is 15 degrees. What is the distance of the boat from the foot of the light house?

Step 1: **Diagram**



OA is the height of the light house

B is the position of the boat

OB is the distance of the boat from the foot of the light house

Step 2: Trigonometric Equation

$$\tan 15^0 = \frac{OA}{OB}$$

Step 3: Solve the equation

$$\tan 15^0 = \frac{60m}{OB}$$

$$OB = 60\text{m} \div \tan 15^0 = 223.92m$$

Question 13

a) If your distance from the foot of the tower is 200m and the angle of elevation is 40^0, find the height of the tower.

b) A ship is 130m away from the centre of a barrier that measures 180m from end to end. What is the minimum angle that the boat must be turned to avoid hitting the barrier?

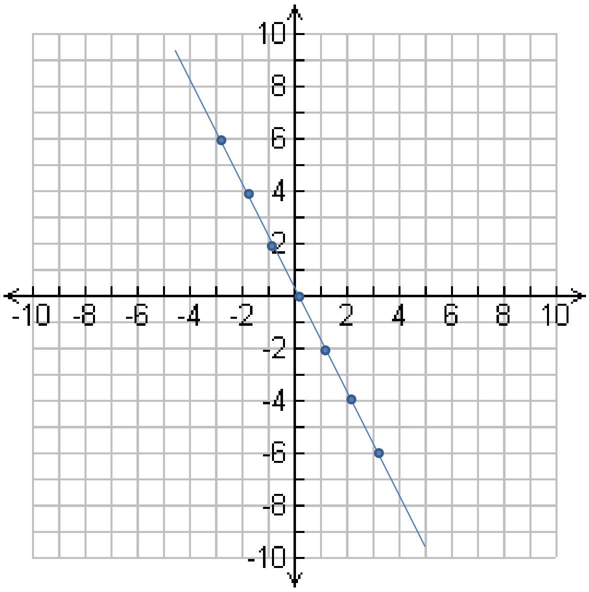
c) Two students want to determine the heights of two buildings. They stand on the roof of the shorter building. The students use a clinometer to measure the angle of elevation of the top of the taller building. The angle is 44^0. From the same position, the students measure the angle of depression of the base of the taller building. The angle is 53^0. The students then measure the horizontal distance between the two buildings. The distance is 18.0m. How tall is each building?

9. Graphs

Linear Patterns

Cartesian Planes are often used to plot data points to see if there is a pattern in the data. A common method is using a table like the one below; the coordinates are plotted on the graph.

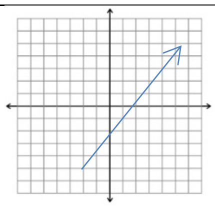
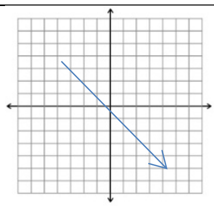
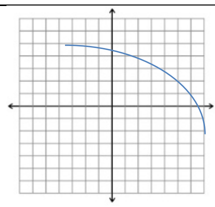
x value	-3	-2	-1	0	1	2	3
y value	6	4	2	0	-2	-4	-6
Coordinate	(-3, 6)	(-2, 4)	(-1, 2)	(0, 0)	(1, -2)	(2, -4)	(3, -6)



- From the graph, we can describe the relationship between the x values and the y values.
  - Because a straight line can be drawn through each point, we can say that there is a **linear** relationship in the data.
  - The graph goes 'downhill' from left to right, which implies the relationship is negative.

  So we can deduce that the relationship is **a negative linear relationship.**

Summary:

		
Positive Linear Relation	Negative Linear Relationship	Non Linear Relationship

Gradient KHANACADEMY Watch this short Khan Academy video for further explanation: **"Plotting (x,y) relationships"**

https://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing_solutions2/v/plotting-x-y-relationships

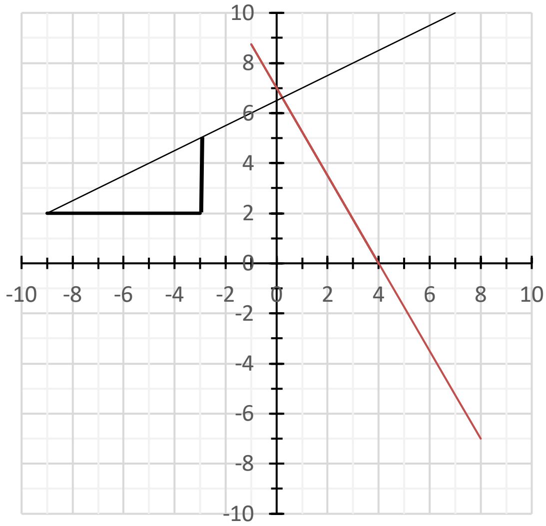
Looking at the two lines plotted on the graph below, we can see that one line is steeper than the other; this steepness is called gradient and has the symbol *m*.

The formula to calculate gradient is:

$$gradient = \frac{difference\ in\ y\ coordinates}{difference\ in\ x\ coordinates}$$

$$\therefore \qquad m = \frac{\Delta y}{\Delta x}$$

Often simplified to: $m = \frac{Rise}{Run}$



To use the formula we draw a right angled triangle on the line (trying to use easy values to work with).

On this triangle we have marked two y coordinates at 5 and 2; and two x coordinates at $-3, -9$.

Thus, looking at the coordinates marked by the triangle, we can say that there is a:

- **rise** of 3: $difference\ in\ y\ coordinates\ (5 - 2 = 3)$ and
- **run** of 6: $difference\ in\ x\ coordinates\ -3 - (-9) = 6$

so $m = \frac{Rise}{Run} = \frac{3}{6} = \frac{1}{2} = 0.5$

∴ The solid line has a positive gradient of 0.5. (It is positive because it is 'uphill'.)

Question 14:

a) Calculate the gradient of the red line.

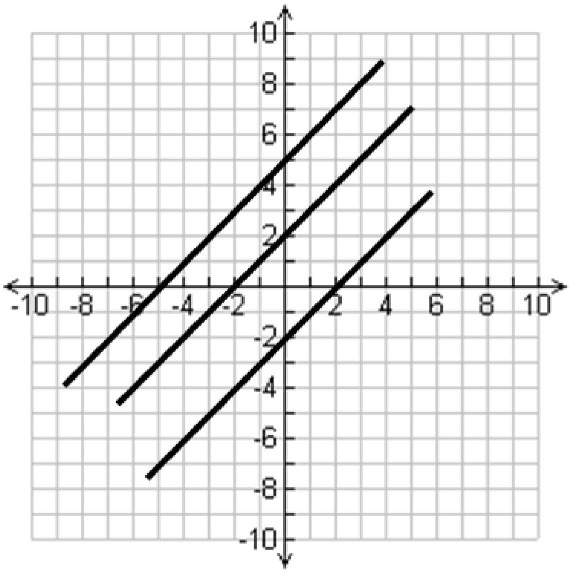
$$gradient = \frac{difference\ in\ y\ coordinates}{difference\ in\ x\ coordinates}$$

Watch this short Khan Academy video for further explanation:

"Slope of a line"
https://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/slope-and-intercepts/v/slope-of-a-line

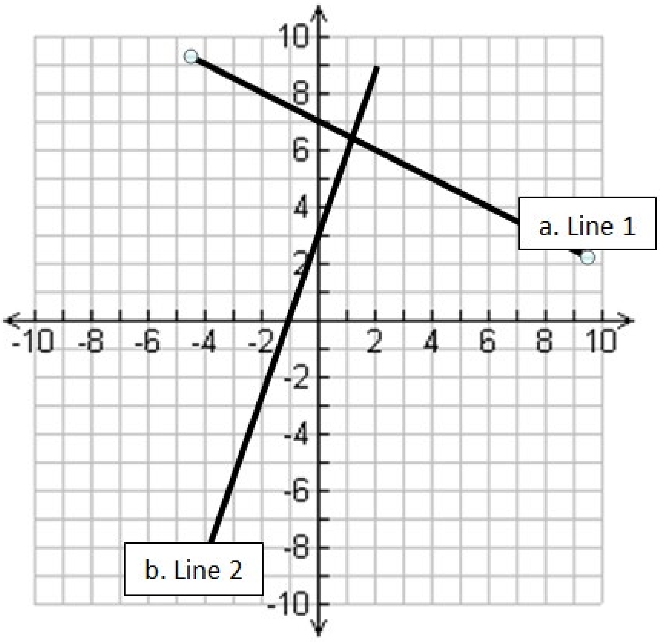
Intercept

- Look at the three lines below, they each have the same slope, yet they are obviously different graphs. To describe how the lines differ from each other, we describe the y intercept, which is given the symbol c.
- The y intercept is the point where the line passes through the y axis. The lines below have y intercepts of 5, 2 and -2.



Question 15:

Calculate the gradient and y intercept for the two lines, a and b, below.



Watch this short Khan Academy video for further explanation:
https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:forms-of-linear-equations/x2f8bb11595b61c86:intro-to-slope-intercept-form/v/slope-intercept-form

Now that we can calculate the gradient and y intercept, we can show the equation for a linear graph. All linear graphs have the following format:

$$y = mx + c$$

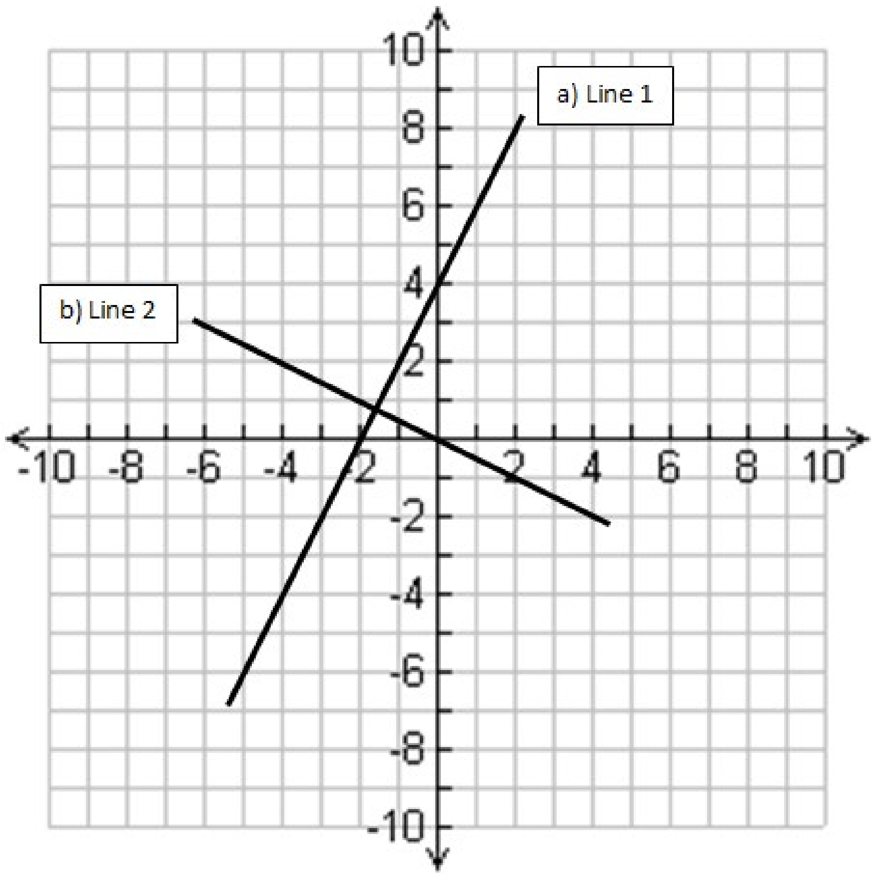
- So the equation: $y = 2x + 3$ has a positive gradient of 2 and has a y intercept of 3.
- From Question 20, we can now show that the two lines have the equations:
  - $y = 3x + 3$
  - $y = -0.5x + 7$
- Now that we have the equation we can use it to predict points.
  - For the line $y = 3x + 3$, when x equals 12, we substitute x for 12, to get $y = 3 \times 12 + 3 \therefore y$ must equal 39.

Question 16:

Write the equation for each of the lines below and calculate the y value when $x = 20$.

a)

b)



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Watch this short Khan Academy video for further explanation:
"Constructing equations in slope-intercept form from graphs"
https://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/equation-of-a-line/v/graphs-

Graphing Equations

So far our focus has been on linear graphs, but some equations produce curved graphs.

Example Problems: Show the following lines on a graph and comment on the shape. Create a value table first.

1. $y = 2x$ 2. $y = x^2$ 3. $y = \frac{1}{x}$

1. $y = 2x$

X value	-3	-2	-1	0	1	2	3
Y value	-6	-4	-2	0	2	4	6
Coordinate	(-3, -6)	(-2, -4)	(-1, -2)	(0, 0)	(1, 2)	(2, 4)	(3, 6)

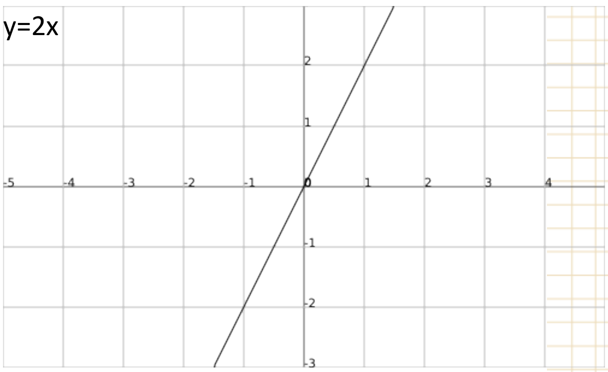
2. $y = x^2$

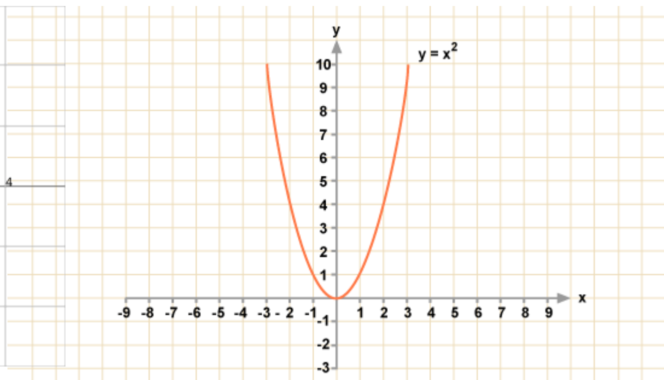
X value	-3	-2	-1	0	1	2	3
Y value	9	4	1	0	1	4	9
Coordinate	(-3, 9)	(-2, 4)	(-1, 1)	(0, 0)	(1, 1)	(2, 4)	(3, 9)

3. $y = \frac{1}{x}$

X value	-3	-2	-1	0	1	2	3
Y value	-0.33	-0.5	-1	error	1	.5	.33
Coordinate	(-3, -0.33)	(-2, -0.5)	(-1, -1)		(1, 1)	(2, .5)	(3, .33)

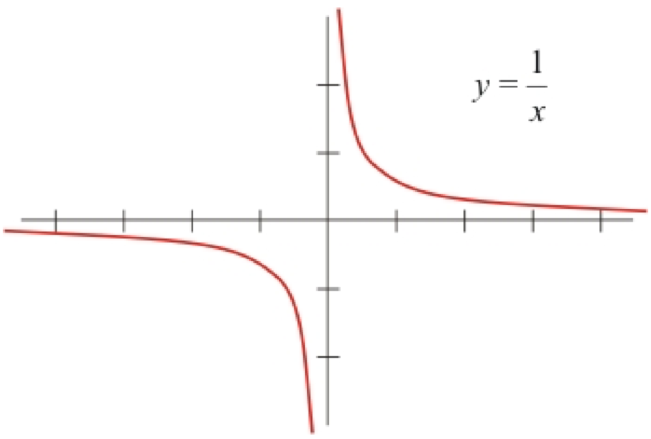
Step 2: Plot the graph



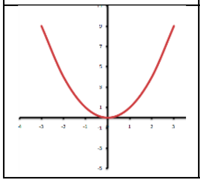
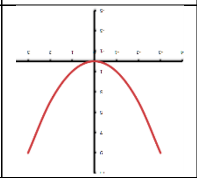
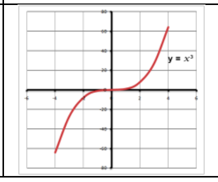
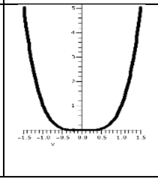
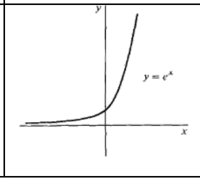
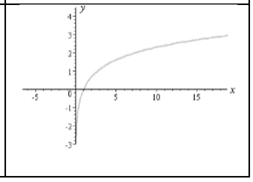


Step 3: Comment on the shape

1. Positive Linear
2. Non Linear (Parabola)
3. Non Linear (does not touch zero)



Common curve shapes:

x^2	$-x^2$	x^3	x^4	e^x	$\ln x$
					

Question 17:

On the graph below, plot the following three equations and comment on the shape.

a) $y = 3x - 2$
b) $y = x^2 + 1$
c) $y = -x^2 + 4$

a)

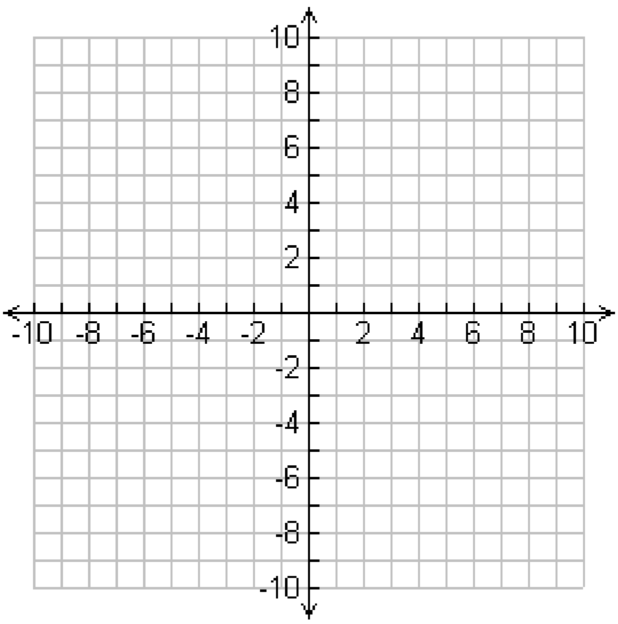
X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							

b)

X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							

c)

X value	-3	-2	-1	0	1	2	3
Y value							
Coordinate							



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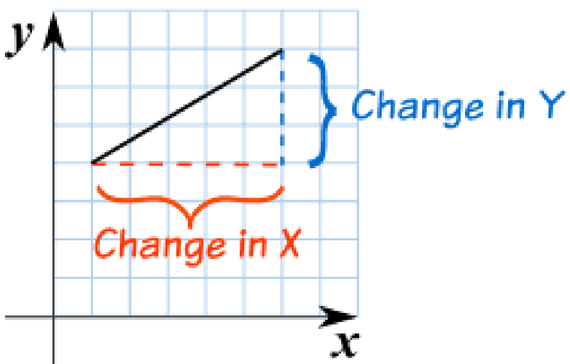
Watch this short Khan Academy video for further explanation:
"Graphs of linear equations"
https://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing_solutions2/v/graphs-of-linear-equations

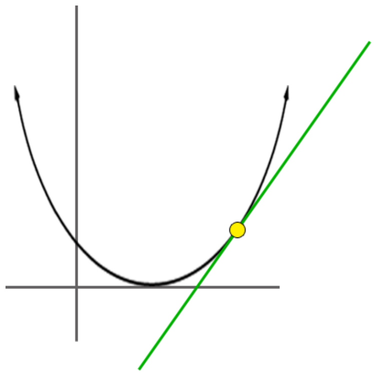
The Gradient of a Curve

In the previous section when calculating the gradient of a straight line, we used the formula $m = \frac{\Delta y}{\Delta x}$. Hence, at any point on the line, the gradient remains the same. The example below left, always has a gradient of $+0.6$.

A curve is different; the slope of a curve will vary at different points. One way to measure the slope of a curve at any given point is to draw a **tangent**. The tangent is a straight line that will just touch the curve at the point to be measured. An example is shown in the image below right; the tangent drawn shows the gradient at that point. If the point was moved either left or right, then the angle of the tangent changes.

- The gradient of the tangent at that point equals the gradient of the curve at that point.





Example problem: Let's use the information from 18. b. and the plot you have drawn.

1. Plot a graph for $y = x^2 + 1$, for values of x between -3 and 3; hence, we created a table of values:

X value	-3	-2	-1	0	1	2	3
Y value	10	5	2	1	2	5	10
Coordinate	-3,10	-2,5	-1,2	0,1	1,2	2,5	3,10

2. Draw in a tangent A at (1,2)

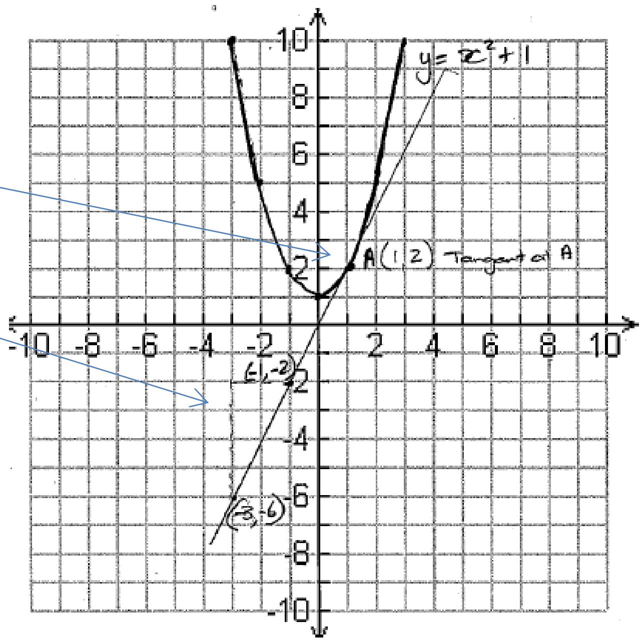
3. Calculate the gradient of the tangent (drawn by eye at this stage)and find the gradient of the curve at A.

Thus we select two points on the tangent which are $(-1, -2)$ and $(-3, -6)$

Now calculate the gradient of the tangent: $m = \frac{\Delta y}{\Delta x}$

which is: $gradient = \frac{difference\ in\ y\ coordinates}{difference\ in\ x\ coordinates}$

$m = \frac{-2-(-6)=4}{-1-(-3)=2} = 2$ ∴ the gradient of the tangent is 2 and thus the gradient of the curve at point A(1,2) is 2.



Question 18:

Using the information above, calculate the gradient of a tangent at B $(-2,5)$.

10. Answers

Q 1. Order of Operations

a) 1
b) -10
c) 12
d) 14
e) 5

Q2. Percentage

a) 7% b) 54% c) 7395 Kj d) 430.1 N

Q3. Collecting like terms

a. 2x + 2y b. 2m + 5n -7 c. $-3x^3 - x^2 + 2x$ d. 11m + 10n e. 10x + 22

Q4. Solving equations

a. x = 15 **b.** x = 16 **c.** x = 9 **d.** x = 14

Q5.

a. x = 7 **b.** x = 162 **c.** x = -242 **d.** $\overrightarrow{v_1} = \overrightarrow{v_2} - \vec{a}t$ **e.** $\frac{\overrightarrow{v_2}^2 - \overrightarrow{v_1}^2}{2\vec{a}} = \vec{x}$

Q6. Rearranging formulas

a. i. 25 ii. 3 iii. 400 **b.** i. 48 ii. 27 iii. 4 **c.** i. 10 ii. 6

d. i. 5 ii. 31 **e.** i. 200 ii. 3 **f.** i. 15 ii. 6 iii. 5

Q7. Power Operations

a)

i. $5^2 \times 5^4 + 5^2 = 5^6 + 5^2$

ii. $x^2 \times x^5 = x^7$

iii. $4^2 \times t^3 \div 4^2 = t^3$

iv. $(5^4)^3 = 5^{12}$

v. $\frac{2^4 3^6}{3^4} = 2^4 3^2 = 16 \times 9 =$ 144

vi. $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{27}$

vii. $\frac{9(x^2)^3}{3xy^2} = \frac{9x^6}{3xy^2} = \frac{3x^5}{y^2}$

viii. $a^{-1}\sqrt{a} = a^{-1} \times a^{\frac{1}{2}} = a^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{a}}$ or $\frac{1}{a^{\frac{1}{2}}}$

b)

i. $x = 2$

ii. $x = -2$

iii. $x = 3$

iv. $x = 5$

v. Show that $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$

$$\frac{\cancel{16}a^2b^3}{\cancel{3}a^3b} \times \frac{\cancel{9}a^3b^5}{\cancel{8}b^2a}$$

$$= \frac{2a^5b^8 \times 3}{b^3a^4} = \frac{2a^1b^5 \times 3}{1} = 6ab^5$$

Q8 Working with units

a) 1.2 kg
b) $4.264 \times 10^{-3}\ kL$ & 4264 mL
c) $6.7 \times 10^{-4}\ g$
d) 1000000 g (or, 1×10^{6} g); 1000 kg
e) 15 inches
f) 114 000 cm
g) 720 seconds

Q9.

a) 250 000 kg/m^3
b) 2 m/s
c) 60 N

Q10 Using Pythagorean theorem

a). 17

b) 7.94

c) $25m$

d) i) $6cm$ ii) $24cm^2$

e). ΔABC is a right angle triangle because $21^2 + 20^2 = 29^2$

whereas ΔDEF is not because $7^2 + 24^2 \neq 27^2$

f) $\angle A$ $(\angle BAC)$ is a right angle

Q11.

a) 1.26 b) 126°

Q12 Trigonometric functions

a) $\tan a = \frac{8}{15}$ $a \approx 28^0$ Note: to calculate the angle on your calculator, press “Shift” – “tan($\frac{8}{15}$)"

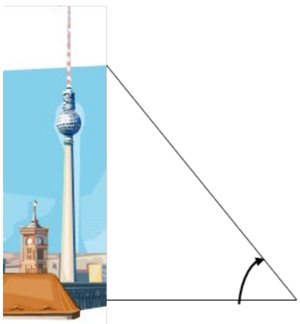
$\tan b = \frac{15}{815}$ b $\approx 62^0$

b) $\sin 60^0 = \frac{c}{10}$ $c \approx 8.66$

c) $\cos 28^0 = \frac{x}{55}$ $x \approx 48.56$

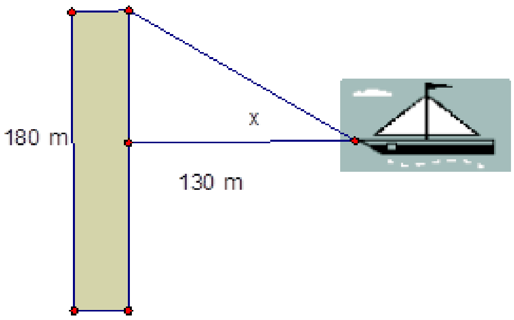
Q13 Solving trigonometric problems

a)



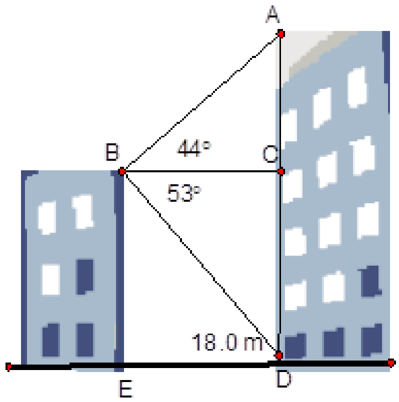
$\tan 40^0 = \frac{h}{200m}$ $h \approx 167.82m$

b)



$\tan x = \frac{90m}{130m}$ $\quad x \approx 34.70^0$

c.



<u>left building</u>:

90⁰ - 53⁰ = 37⁰

$\tan 37^0 = \frac{18m}{BE}$ $\quad$ BE$\approx 23.89m$

<u>right building</u>:

$\tan 44^0 = \frac{AC}{18m}$ $\quad$ AC$\approx 17.38m$

AD = BE + AC = 23.89m + 17.38m = 41.27m

Q14. Gradient

a. $-\frac{7}{4} = -1\frac{3}{4} = -1.75$

Q15. Intercept

a. Line 1: $m = -0.5, c = 7$ $\qquad$ b. Line 2: $m = 3, c = 3$

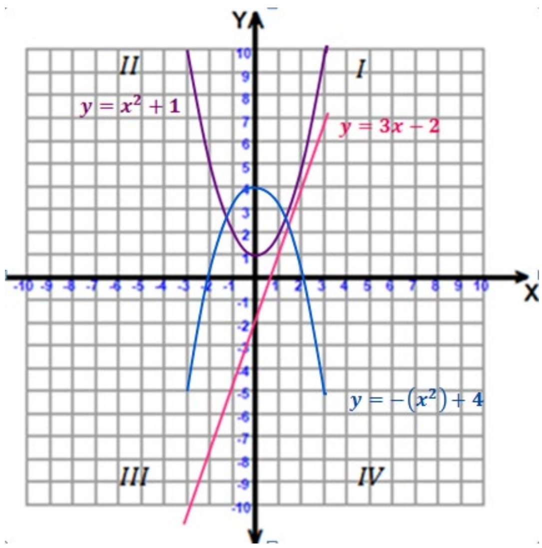
Q16. Linear Equation

a. Line 1: $y = 2x + 4 \;\therefore y = 44 \; when \; x = 20$

b. Line 2: $y = -\frac{1}{2}x + 0 \;\;\therefore y = -10 \; when \; x = 20$

Q17. Graphing Equations

a. Positive Linear $\qquad$ b. U Shaped $\qquad$ c. ∩ Shaped



Q18. Tangent

a. -4