Maths Refresher

Simplifying Equations

Learning, Teaching
and Student Engagement

Simplifying Equations

Learning intentions ....

- Algebra
- Order of operations
- Commutative property
- Associative Property
- Distributive property
- Simplify with grouping symbols

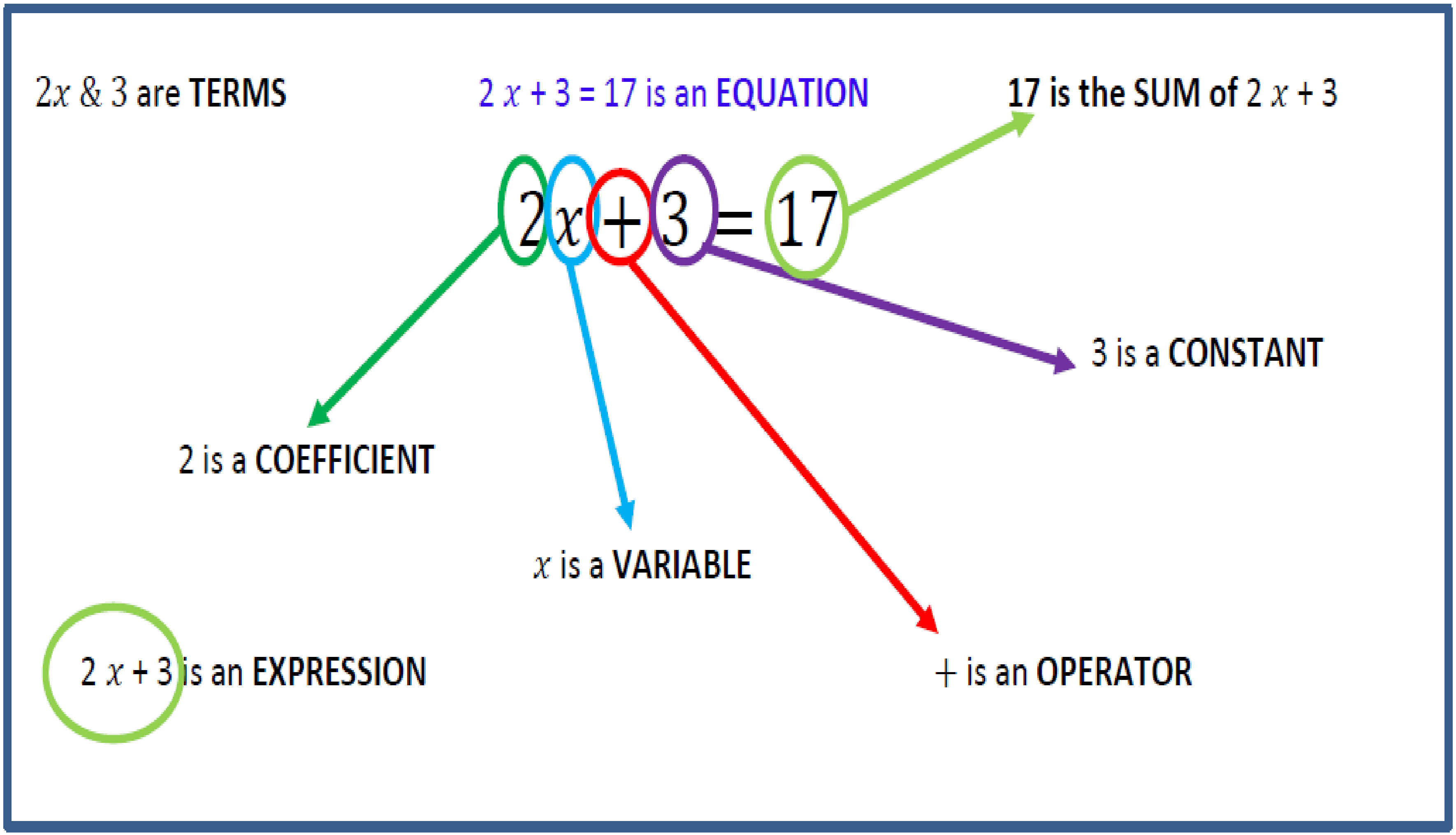
What is algebra

- Algebra has many definitions; some adults have grown up believing that algebra is somewhat scary and difficult.
- Algebra involves finding and **communicating** number patterns and relationships
- As number patterns become more complex they are more difficult to communicate verbally…
- Hence, notation is used to simplify the task.
- Algebra is actually a very useful and simple concept, the complex part is familiarising yourself with the language.
- The simplest definition for algebra is:
  - **A mathematical method for finding an unknown number.**

Glossary

- **Equation**: Is a mathematical sentence. It contains an equal sign meaning that both sides are equivalent.
- **Expression**: An algebraic expression involves numbers, operation signs, brackets/parenthesis and pronumerals that substitute numbers.
- **Operator**: The operation $(+\ ,-\ ,\times\ ,\div)$ which separates the terms.
- **Term**: Parts of an expression separated by operators.
- **Pronumeral:** A symbol that stands for a particular **value**.
- **Variable**: A letter which represents an **unknown** number. Most common is x, but it can be any symbol.
- **Constant**: Terms that contain only numbers that always have the same value.
- **Coefficient**: Is a number that is partnered with a variable. Between the coefficient and the variable is a multiplication. Coefficients of 1 are not shown.

Glossary



$2x$ & 3 are TERMS
$2\ x + 3 = 17$ is an EQUATION
17 is the SUM of $2\ x + 3$
$2x + 3 = 17$
3 is a CONSTANT
2 is a COEFFICIENT
x is a VARIABLE
$2\ x + 3$ is an EXPRESSION
$+$ is an OPERATOR

Glossary example

Pronumeral:	x	Operator:	$+$
Variable:	x	Term:	3 & $2x$ ($2x$ is a term with 2 factors)
Constant:	3		
Equation:	$2x + 3 = 17$	Left hand expression:	$2x + 3$
Coefficient:	2	Right hand expression	17 (which is the sum of the LHE)

Some algebra rules …

Expressions with zeros and ones

- Zeros and ones can be eliminated, why:
- When we add zero it does not change the number, $x + 0 = x$
- If we multiply by one, then the number stays the same, for example: $x \times 1 = x$
- What we do to one side we do to the other
- …and the BODMAS rule

B	Brackets first
O	Orders (ie Powers and Square Roots, etc.)
DM	Division and Multiplication (left-to-right)
AS	Addition and Subtraction (left-to-right)

Order of Operations

Revision: Example 1	Revision: Example 2	Revision: Example 3
$50 - 3 \times 2 \times 5 =$ $50 - 6 \times 5 =$ $50 - 30 =$ 20	$10 + 2 - 3 \times 4 =$ $10 + 2 - 12 =$ $12 - 12 =$ 0	$32 \div 2 - 2 \times 3 =$ $16 - 2 \times 3 =$ $16 - 6 =$ 10

Your turn ….

ORDER OF OPERATIONS (Muschla et al., 2011)

1. $12 - 2 \times 4 + 1$

2. $12 \times 4 \div 2 - 3$

3. $10 \times 2 - 2 \times 8$

4. $10 \times 2 - 6 \div 3$

5. $8 + 1 + 6 \times 5 \div 2$

6. $48 \div 2 \times 8 - 4$

7. $15 - 2 - 2 \times 6$

8. $35 + 8 - 12 \div 3$

9. $3 \times 7 - 8 - 5$

10. $20 \times 2 + 10 - 4 \div 2$

11. $8 - 4 + 2 \times 3$

12. $40 \div 8 - 5 + 3 \times 2 + 10$

What is the missing 'operation' symbol? $12 \boxed{} 3 \times 2 + 2 = 8$

Answers

1. $12 - 2 \times 4 + 1$

$12 - 8 + 1 =$

$4 + 1 =$

5

2. $12 \times 4 \div 2 - 3$

$48 \div 2 - 3 =$

$24 - 3 =$

21

3. $10 \times 2 - 2 \times 8$

$20 - 16 =$

4

4. $10 \times 2 - 6 \div 3$

$20 - 2 =$

18

5. $8 + 1 + 6 \times 5 \div 2$

$8 + 1 + 30 \div 2 =$

$8 + 1 + 15 =$

24

6. $48 \div 2 \times 8 - 4$

$24 \times 8 - 4 =$

$192 - 4 =$

188

7. $15 - 2 - 2 \times 6$

$15 - 2 - 12 =$

$13 - 12 =$

1

8. $35 + 8 - 12 \div 3$

$35 + 8 - 4 =$

$43 - 4 =$

39

9. $3 \times 7 - 8 - 5$

$21 - 8 - 5 =$

8

10. $20 \times 2 + 10 - 4 \div 2$

$40 + 10 - 2 =$

48

11. $8 - 4 + 2 \times 3$

$8 - 4 + 6 =$

10

12. $40 \div 8 - 5 + 3 \times 2 + 10$

$5 - 5 + 6 + 10 =$

16

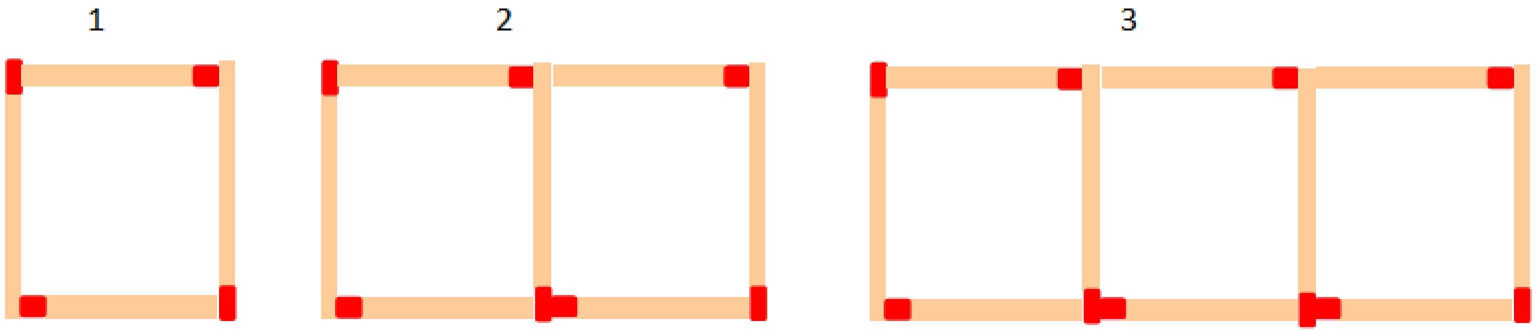
What is the missing 'operation' symbol: $12 - 3 \times 2 + 2 = 8$

$-$

Some algebra rules

- **Multiplicative Property:** $\mathbf{1 \times x = x}$
  - Multiplying any number by one makes no difference.
- **Additive Inverse:** $\mathbf{x + (-x) = 0}$
  - Any number added to its negative equals zero.
- **Multiplicative Inverse:** $\mathbf{x \times \frac{1}{x} = 1}$
  - Any number multiplied by its reciprocal equals one.
- **Symmetric Property:** $\mathbf{x = y \; then \; y = x}$
  - Perfect harmony!
- **Transitive Property:** $\mathbf{If \; x = y \; and \; y = z \; then \; x = z}$
  - For example, if apples cost \$2 and oranges cost \$2 then apples and oranges are the same price.

Understanding an algebraic expression



Let’s investigate this pattern

- The first thing we do is number each element, if you do not number the element you will not be able to see the relationship between the ‘element number’ (which is the ‘term’) and the ‘total number of sticks’ in each element. If you cannot see the relationship, how the pattern **changes,** you will not be able to work out a formula.

...continued

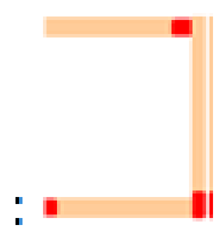
Consider:

a)What is **changing** in this pattern

b) What is the repeating part?

c) What stays the same?

- Each time it grows by 3 sticks.



- The first stick remains the same.

Number of elements or term	Total number of sticks
T_1	4
T_2	7
T_3	10
T_4	13
T_n	y

…continued

- Each new term grows by three, so for each term – step in the pattern – another ‘group’ of three is added.
- However, there is always one matchstick that stays the same – ‘the constant’
- Therefore, the generalisation (general rule) or ‘algebraic equation’ for the matchstick pattern would be:
  - $n \times 3 + 1 = y$
  - $3n + 1 = y$
- (The number of elements (the term) times three plus one = the total number of matchsticks)

Commutative property

- Think of the term ‘commutative’ in relation to being able to move things around – to commute.
- Hence, **Commutative Property** is the property where we can move things around
  - **The Commutative Law of Addition:**

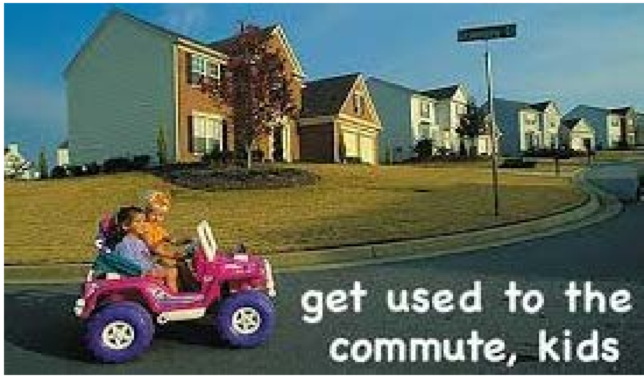
$$x + y = y + x$$

For example, 2+3 = 3+2

  - **The Commutative Law of Multiplication:**

$$x \times y = y \times x$$

For example, 2 × 3 = 3 × 2



Associative Property

- **The Associative Law of Addition:**

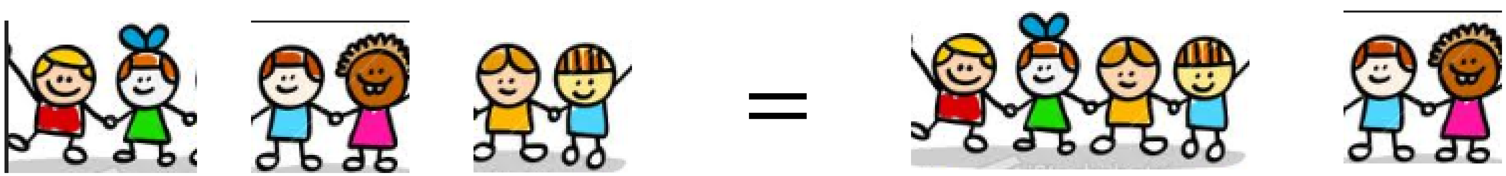
$$(x + y) + z = x + (y + z)$$

The order you add numbers does not matter. The difference is that we **‘regroup’** the numbers

- **The Associative Law of Multiplication:**

$$(x \times y) \times z = x \times (y \times z)$$

The order you multiply numbers does not matter. The difference is that we **‘regroup’** the numbers, whereas in commutative property the numbers are **moved** around – **not** regrouped.



Distributive property

- **The Distributive Law:** multiplication **distributes** *over* addition or subtraction *through* the brackets (parentheses) $\boldsymbol{x(y+z) = xy + xz}$

For example, $2(3+4) = 2 \times 3 + 2 \times 4$

$$2(7) = 6 + 8$$

$$14 = 14$$

Your turn …

MULTIPLICATION AND ADDITION PROPERTIES

Practise problems:

1. Rewrite $3 \times 2 \times x$ by using the 'commutative property' and then simplify

2. Rearrange $2(4x)$ by using the 'associative property' and then simplify

3. Rewrite $8(2 + x)$ using the 'distributive property' and then simplify

Answers

- Rewrite $3 \times 2 \times x$ by using the 'commutative property'

  $\mathbf{3 \times 2x}$ $\boldsymbol{or}$ $\mathbf{2 \times 3x}$ $\mathbf{6x}$ **(simplified).**

- Rearrange $2(4x)$ in using the 'associative property'

  $8 \times \boldsymbol{x}$ $\mathbf{8x}$ **(simplified)**

- Rewrite $8(2 + x)$ using the 'distributive property'

  $\mathbf{(8 \times 2) + (8x)}$ $\mathbf{16 + 8x}$ **(simplified)**

Collecting ‘like’ terms

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Collect like terms

$$4a + 5 + 2a - 3$$

$$= 6a + 2$$

KHANACADEMY

Watch this short Khan Academy video for further explanation:
“Combining like terms, but more complicated”
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-variables-expressions/cc-7th-manipulating-expressions/v/combining-like-terms-3

Like terms

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Often real life algebra problems look like the following:

- $7x + 2x + 3x - 6x + 2 = 14$

- It is difficult to even try to start solving a problem so large. What we need to do is simplify the problem into a smaller problem. We do this by collecting **like** terms.

- A **like** term is a term which has the same variable to the same power only the **coefficient** is different.

- Looking at the example: $7x + 2x + 3x - 6x$ all **coefficients** have the same **variable**.

- We can treat them as a simple equation: $7 + 2 + 3 - 6$ which equals 6, so:

$$7x + 2x + 3x - 6x = 6x$$

$$\therefore \quad 6x + 2 = 14$$

Like terms

Combine like terms.

A. $2x + 5x - 4y + 3$

$2x + 5x - \boxed{4y} + \boxed{3}$ — *Identify like terms.*

$7x - 4y + 3$ — *Combine coefficients: 2 + 5 = 7*

B. $9d + 7c - 4d - 2c$

$9d + \boxed{7c} - 4d - \boxed{2c}$ — *Identify like terms.*

$5d + 5c$ — *Combine coefficients: 9 – 4 = 5 and 7 – 2 = 5*

C. $8g + c - 6$

$\boxed{8g} + c - \boxed{6}$ — *No like terms.*

KHANACADEMY

Watch this short Khan Academy video for further explanation:
"Combining like terms and the distributive property"
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-variables-expressions/cc-7th-manipulating-expressions/v/combining-like-terms-and-the-distributive-property

Like terms

Collect the like terms and simplify:

$$5x + 3xy + 2y - 2yx + 3y^2$$

Step 1: Recognise the like terms:

$(xy\ is\ the\ same\ as\ yx\ -\ see\ the\ commutative\ law)$

$$5x + 3\boldsymbol{xy} + 2y - 2\boldsymbol{yx} + 3y^2$$

Step 2: Arrange the expression so that the like terms are together (Remember to take the operator with the term)

$$5x + 2y + 3\boldsymbol{xy} - 2\boldsymbol{yx} + 3y^2$$

Step 3: Simplify

$$5x + 2y + \boldsymbol{xy} + 3y^2$$

Your turn …

SIMPLIFY BY COLLECTING LIKE TERMS:

a. $3m + 2n + 3n - m - 7 =$

b. $4(x + 7) + 3(2x - 2) =$

c. $3(m + 2n) + 4(2m + n) =$

d. $\frac{x}{3} + \frac{x}{4} =$

Answers

$a.\ \ 3m + 2n + 3n - m - 7 = 2m + 5n - 7$

$$a.\ \ 4(x+7) + 3(2x-2) = 4x + 28 + 6x - 6$$
$$= 10x + 22$$

$$c.\ \ 3(m+2n) + 4(2m+n) = 3m + 6n + 8m + 4n$$
$$= 11m + 10n$$

$$d.\ \ \frac{x}{3} + \frac{x}{4} = \frac{4x+3x}{12} = \frac{7x}{12}$$

Simplify with grouping symbols

Use Distributive Rule

What is the answer to **2(4 + 3)** ?

$$2\,(4 + 3) = 2 \times 4 + 2 \times 3 = \mathbf{14}$$ ✓

The "2" outside the brackets is multiplied onto everything that is inside the brackets.

Watch this short Khan Academy video for further explanation:
"Factoring and the distributive property 2"
https://www.khanacademy.org/math/algebra-basics/quadratics-polynomials-topic/Factoring-simple-expressions-core-algebra/v/factoring-and-the-distributive-property-2

Simplify with nested grouping symbols

When there are **two** sets of brackets – one is *nested* inside the other – operations in the inner set must be worked first.

KHANACADEMY

Watch this short Khan Academy video for further explanation:
"Expression terms, factors and coefficients"
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-expressions-and-variables/cc-6th-writing-expressions/v/expression-terms-factors-and-coefficients

Simplify with nested grouping symbols

Example 1	Example 2	Example 3
$20 - [3 \times (14 - 12)]=$ $20 - [3 \times 2] =$ $20 - 6 =$ 14	$4[(6 + 3) \times 10] =$ $4[9 \times 10] =$ $4(90) =$ 360	$\frac{3 + 24}{12 - (10 - 7)} =$ $\frac{3+24}{12-3} =$ $\frac{27}{9} =$ 3

Your turn …

SIMPLIFYING EXPRESSIONS WITH NESTED GROUPING SYMBOLS

Simplify the following: (Muschla et al., 2011)

1. $4[9 - (5 - 2)]$
2. $2[4 + 7(4 - 3)]$
3. $2[3(12 - 7) \times 4]$
4. $4[8(6 - 3) - 4]$
5. $3 + 2[4(3 + 8)]$
6. $(3 + 2)[4(3 + 8)]$

Answers

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1. $4[9 - [5 - 2]]$

$4(9-3) =$

$4(6) = 24$

2. $2[4 + 7[4 - 3]]$

$2(4+7\times 1) =$

$2\times 11 = 22$

3. $2[3[12 - 7] \times 4]$

$2(3\times 5\times 4) =$

$2\times 60 = 120$

4. $4[8[6 - 3] - 4]$

$4(8\times 3-4) =$

$4\times 20 = 80$

5. $3 + 2[4[3 + 8]]$

$3+2(4\times 11) =$

$3+2\times 44 = 91$

6. $[3 + 2][4[3 + 8]]$

$(3+2)\times(4\times 11) =$

$5\times 44 = 220$

Simplify with grouping symbols

- Now let's work with an algebraic expression with brackets and simplify by *removing* the brackets and including powers.
- Recap:
  - So if we multiply two numbers together, the order in which we multiply is irrelevant – **commutative property**
  - Simplify $4(3x)$
  - This could be written as $4 \times (3 \times x)$
  - And then as $(4 \times 3) \times x$
  - Therefore, we can simplify to $12x$

Simplify with grouping symbols and exponents

Another example incorporating **exponents**:

- Simplify $(3x)(6x)$
- $(3 \times x) \times (6 \times x)$
- We can change the order $(3 \times 6) \times (x \times x)$
- Therefore, we can simplify to $18x^2$

Simplify with grouping symbols and exponents

- Remember we need to follow the order of operations rule BODMAS
- …and now we also apply the 'INDEX LAWS' from the previous session

Simplify with grouping symbols and exponents

Is $2x^2$ the same as $(2x)^2$?

- $2x^2$ is $2 \times x \times x$
  - $so\ if\ x\ was\ 5$
  - $2 \times 5 \times 5 = 50$
- $(2x)^2$ is $(2 \times x) \times (2 \times x)$
  - $so\ if\ x\ was\ 5$
  - $10 \times 10 = 100$

Simplify with grouping symbols and exponents

This example explains how any order property can be used:

$$\begin{aligned} 6x \times 2y \times 3xy &= 6 \times x \times 2 \times y \times 3 \times x \times y \\ &= 6 \times 2 \times 3 \times x \times x \times y \times y \\ &= 36x^2y^2 \end{aligned}$$

Simplify with grouping symbols and exponents

Simplify $5x^2 \times 6x^5$

- So we can say that we have
  - $(5 \times 6) \times (x \times x) \times (x \times x \times x \times x \times x)$
  - Which is $30 \times x^{2+5}$
  - Therefore, $30x^7$

(Remember the first index law from last week.)

Simplify with grouping symbols and exponents

- Simplify
  - What is the difference
  - $(6x)(5x) \quad and \quad (6x) + (5x)$

    $30x^2$ $\qquad$ $11x$ (Here we **add** like terms)
- One more, are these the same:
  - $(-5a^2)(-2a)$
  - $10a^3$
    - $(-5) \times (-2) = 10$
    - $a^2 \times a^1 = a^3$(index law one)
    - Therefore, $(-5a^2)(-2a)$ is the same as $10a^3$

Your turn …

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SIMPLIFY WITH GROUPING SYMBOLS AND EXPONENTS

Practise problems:

1. $3x + 6x + 11x =$
2. $3xy + 8xy =$
3. $5x^2 - 4x^2 =$
4. $5x^2 + 6x + 4x =$
5. $7x^2y + 3x^2y + 6xy =$

Answers

1. $3x + 6x + 11x = 20x$
2. $3xy + 8xy = 11xy$
3. $5x^2 - 4x^2 = x^2$
4. $5x^2 + 6x + 4x = 5x^2 + 10x$
5. $7x^2y + 3x^2y + 6xy = 10x^2y + 6xy$

Simplify systematically

Steps to follow which may assist your reasoning process

1. Simplify expressions that have grouping symbols first and work from the innermost to the outer. As you do this apply the BODMAS rule too.
2. Simplify powers
3. Multiply in order from left to right
4. Add and subtract in order from left to right.
5. Then work backwards to check

Your turn …

SIMPLIFY EXPRESSIONS WITH GROUPING SYMBOLS AND EXPONENTS (Muschla et al., 2011)

1. $2 \times 3^2 - 5$

2. $37 + 5^2 \times 3$

3. $2^3 + 7(8 - 3)$

4. $(2 + 1)^3 - 2^3$

5. $(13 - 2^3) \times 2 + 6^2$

6. $2^2 \times 8 - 5 - 1$

7. $\dfrac{48}{2^4}$

8. $\dfrac{(5 - 2)^3}{2 \times 7 - 5}$

Answers

1. $2 \times 3^2 - 5 =$

$2 \times 9 - 5 =$

$18 - 5 =$

13

2. $37 + 5^2 \times 3$

$37 + 25 \times 3 =$

$37 + 75 =$

112

3. $2^3 + 7(8-3)$ $= 2^3 + 7(5) =$

$8 + 35 =$

43

4. $(2+1)^3 - 2^3$

$3^3 - 2^3 =$

$27 - 8 =$

19

5. $(13 - 2^3) \times 2 + 6^2$ $(13-8) \times 2 + 36 =$

$5 \times 2 + 36 =$

46

6. $2^2 \times 8 - 5 - 1$

$4 \times 8 - 5 - 1 =$

$32 - 5 - 1 =$

$27 - 1 =$

26

7. $\frac{48}{2^4} = \frac{48}{16} = 3$

8. $\frac{(5-2)^3}{2 \times 7 - 5}$

$\frac{3^3}{14-5} =$

$\frac{27}{9} = 3$

Your turn …

Simplifying expression with grouping symbols and exponents

…and a challenge:

What is the missing number?

$$\frac{4(\quad+3)^2}{5(14-3^2)}=16$$

Challenge answer

- $\frac{4(\quad +3)^2}{5(14-9)}$=16
- $\frac{4(\quad +3)^2}{5(5)}$=16
- $\frac{4(\quad +3)^2}{25}$=16 so what divided by 25 = 16, $25 \times 16 = 400$
- Now we look at what divided by 4 = 400 which is 100
- What is the square root of 100? 10
- So now we can say that the missing number is 7
- Work backwards to see if this is correct
- $\frac{4(\,7\,+3)^2}{5(14-9)}$=16

Simplifying Equations

Reflect on the learning intentions ....

- Algebra
- Order of operations
- Commutative property
- Associative Property
- Distributive property
- Simplify with grouping symbols

Resources

Australian Mathematical Sciences Institute. (2011). Algebraic expressions. Retrieved from http://www.amsi.org.au/teacher_modules/pdfs/Algebraic_expressions.pdf

Purple Math. (2013). Associative, commutative and distributive properties. Retrieved from http://www.purplemath.com/modules/numbprop2.htm

Muschla, J. A., Muschla, G. R., Muschla, E. (2011). *The algebra teacher's guide to reteaching essential concepts and skills.* San Francisco: Jossey-Bass